One Vol to Rule Them All: Common Volatility Dynamics in Factor Returns*

Nishad Kapadia[1], Matthew Linn[2], and Bradley S. Paye[3]

[1]A. B. Freeman College of Business, Tulane University
[2]Isenberg School of Management, University of Massachusetts at Amherst
[3]Pamplin College of Business, Virginia Tech, Blacksburg, VA 24061, USA

January 15, 2020

ABSTRACT

We show that there is strong commonality in the volatility of a wide range of diversified equity portfolios. Common factor volatility (CFV) exists even when factor or anomaly returns are market-adjusted and does not appear to be attributable to common microstructure noise or a lack of diversification. We show that CFV closely relates to previously identified commonality in *idiosyncratic* volatility, implying that a common volatility feature pervades the entire spectrum of equity return variation. Consistent with the interpretation of CFV as a latent, pervasive equity risk feature, CFV outperforms traditional measures of market volatility in forecasting excess stock market returns. In addition, deviations of factor volatilities from long-run equilibrium relations with CFV forecast innovations in future factor volatility. Several alternative tests indicate only a weak relation between CFV and time-variation in fundamental uncertainty. We also do not find strong support for the hypothesis that variation in growth options, operating or financial leverage drives CFV. The ultimate sources of common equity volatility dynamics therefore constitute an important unresolved puzzle in finance.

*We thank John Y. Campbell, Kim Christensen, Gregory Kadlec, Guilherme Salome and conference and seminar participants at the Fourth International Workshop on Financial Econometrics, the Duke University Financial Econometrics Workshop, Binghamton University, Oklahoma State University, and Virginia Tech for helpful comments. Author contact information: Matthew Linn (mlinn@isenberg.umass.edu), Nishad Kapadia (nkapadi@tulane.edu), and Bradley Paye (bpaye@vt.edu).

Risk occupies a central role in financial economics. Asset pricing theories typically partition equity risk into components attributable to a set of systematic factors and 'idiosyncratic' components that disappear in well-diversified portfolios. While important and interesting along many dimensions, this decomposition obscures the role of dynamic relations among the components of equity risk. This paper presents novel evidence that, in fact, a latent, common volatility feature pervades the entire spectrum of return variation. This common volatility component drives low-frequency time-variation in the volatility of broad stock indices ('the market'), alternative diversified equity factors, long-short anomaly portfolios, and idiosyncratic return components. The presence of a strong common volatility feature has important implications for asset pricing and the modeling and forecasting of equity risk.

To appreciate the essence of our contribution, consider a standard factor model for stock returns:

$$r_{i,t} = \sum f_{j,t}\lambda_j + \epsilon_{i,t}$$

where $r_{i,t}$ is the excess stock return for firm i at time t, λs are factor exposures, and the factors f are orthogonal to each other and to the idiosyncratic return ϵ. The fact that $\epsilon_i \perp \epsilon_k$ does not imply $\epsilon_i^2 \perp \epsilon_k^2$. Indeed, recent literature identifies a common component in the volatility of idiosyncratic returns (see, e.g., Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016)). Similarly, the fact that $f_j \perp f_k$ for $j \neq k$ does not imply that $f_j^2 \perp f_k^2$. In contrast to prior work concerning idiosyncratic returns, our paper assesses the degree of commonality in time-varying volatility among *systematic factors* $f_{j,t}$. We document evidence of common factor volatility across a wide range of equity factors and diversified portfolios. Commonality in volatility does not arise mechanically due to common market exposure and analyses of a range of alternative factor sets produce nearly identical proxies for latent common factor volatility. In addition, the common feature we identify in factor volatility closely relates to the previously identified common component in idiosyncratic return volatility. The evidence therefore indicates that a single common volatility feature drives low-frequency variation across all components of equity risk.

The strong common component we document in factor volatility is not obvious ex ante.

It is not clear, for example, that time-variation in the volatility of the Fama-French size factor should exhibit much commonality with time-variation in the volatility of the momentum factor or the mispricing factor recently proposed by Stambaugh and Yuan (2016). Leading asset pricing theories such as the arbitrage pricing theory (APT) of Ross (1976) and Chamberlain and Rothschild (1983) impose few clear restrictions regarding the nature of co-movement among factor volatilities. From this perspective, the extent of commonality we document in factor volatility is arguably surprising. Common factor volatility has asset pricing implications concerning the properties of the stochastic discount factors (SDFs) associated with conditional factor models. Specifically, strong commonality in factor volatility implies significant time-variation in maximum conditional Sharpe ratios, or, equivalently, a high volatility of the SDF *unless* factor premia also co-move in correlated fashion.[1] Empirically, we show that CFV outperforms traditional measures of market volatility in forecasting excess stock market returns, especially at shorter horizons over which earlier studies often find an insignificant risk-return relation.

Our evidence concerning common volatility variation also has implications concerning the economic sources of time-varying volatility (Schwert (1989), Engle and Rangel (2008), etc.) and applications to the important problem of forecasting volatility. From a forecasting perspective, we show that deviations of factor volatilities from long-run equilibrium relations with CFV forecast innovations in future factor volatility. Similarly, factor volatility forecasting models that include CFV in addition to lagged factor volatility deliver more accurate out-of-sample forecasts relative to benchmarks that exclude CFV. These results suggest that CFV plays a central role in determining the dynamics of equity factor volatility for a wide spectrum of traded factors including the market factor. Turning to the sources of time variation in equity volatility, we test whether the dynamics of common factor volatility appear to be explained by time-variation in economic or fundamental uncertainty. A variety of alternative tests indicate only a weak relation between CFV and fundamental uncertainty.

[1] The presence of strong common factor volatility relates to recent evidence that 'volatility-managed' portfolios produce significant alphas and Sharpe ratio increases due to the fact that changes in volatility are not offset by proportional changes in expected returns (Moreira and Muir (2017)).

The commonality in volatility is much more pronounced in traded equity returns relative to fundamental macroeconomic or cash flow volatility. We also do not find strong support for the hypothesis that variation in growth options, operating or financial leverage drives CFV. The ultimate sources of common equity volatility dynamics therefore constitute an important unresolved puzzle in finance.

Our empirical analysis proceeds as follows. We first extract measures of common volatility across alternative sets of diversified factors, e.g., characteristics-based factors or statistical factors. We consistently find strong evidence of commonality in factor volatility. The common component explains an economically large fraction of variation in factor volatility series. For example, the common volatility component extracted from a broad set of characteristics-based factors explains 70-80% of total variation in factor volatilities. We obtain similar results for a range of alternative factors, including tracking portfolios for macroeconomic factors in the spirit of Chen, Roll, and Ross (1986), industry factors, a large set of long-short anomaly portfolios, and statistical factors in the spirit of Connor and Korajczyk (1986). Moreover, the common factor volatility series extracted from these alternative factor sets are highly correlated, implying that the dynamics of common factor volatility are largely invariant to the particular specification of factors.

We next assess the extent to which there exists a common component in the time-varying *correlations* among factor returns.[2] Here we find substantially weaker results: the first principal component extracted from various sets of factor correlations typically explains only a small fraction of total variation among these correlations. Moreover, in contrast to common factor volatility, common correlation components extracted from different sets of assets differ materially, and common correlation does not strongly correlate with economic conditions. The strong evidence of commonality in factor volatility that we observe, coupled with weak commonality in correlations, suggests that equity factor returns are approximately consistent with the *pure variance common feature* (PVCF) model introduced by Engle and

[2] Given a standard factor model, it is always possible to produce an equivalent set of *unconditionally* uncorrelated factors via rotation. However, this does not preclude the possibility of conditional correlation dynamics among factors. Our analysis explores to what extent there exists commonality in such dynamics.

Marcucci (2006). Under this model, a small number of latent variance features drive all factor volatilities, but the model does not require that the covariances (and specifically correlations) also depend on these same features.[3]

Measures of common factor volatility (CFV) positively correlate with market volatility. This raises the possibility of a relatively uninteresting explanation for observed commonality among non-market factors: perhaps factor and anomaly returns exhibit time-varying market exposure that drives co-movement in factor variances.[4] To address this potential critique, our main results employ volatility proxies constructed using daily factor or anomaly returns that are orthogonalized with respect to market returns. With or without this orthogonalization, we find strong evidence of common factor volatility (CFV), and the time series properties of CFV are similar. Consequently, we conclude that explicit market exposure does not drive the commonality we document. We also reject the hypothesis that CFV spuriously reflects commonality in time-varying market frictions, such as aggregate bid-ask spread variation.

We next consider the relation between common factor volatility and common *idiosyncratic* volatility documented in previous studies (e.g., Herskovic et al. (2016)). It is conceptually possible that CIV generates spurious common factor volatility due to insufficiently diversified factor portfolios. However, we continue to find strong evidence of common factor volatility using broad, equal-weighted factor and industry portfolios. It is unlikely that truly firm-specific return variation contributes appreciably to these portfolio returns. We also consider the converse hypothesis that unmodelled systematic exposures explain apparent commonality in idiosyncratic volatility. To address this conjecture, we extract common volatility measures from alternative portions of the spectrum of return variation based on a statistical factor decomposition. We continue to find strong evidence of commonality in volatility irrespective

[3]Engle and Kozicki (1993) introduce the analysis of common features among time series, generalizing prominent special cases such as cointegration, in which common stochastic trends drive long-run properties of a larger set of time series (Engle and Granger (1987)).

[4]Industry portfolios offer a stark example, as these long-only portfolio returns contain significant market exposure and apparent co-movement in industry return volatility might simply reflect common exposure to (time-varying) market risk. Prominent characteristics-based factor models involve factors constructed as long-short portfolios. This procedure reduces, but does not necessarily eliminate, market correlation. Unconditionally uncorrelated factors can also exhibit nonzero *conditional* correlations.

of whether we examine the leading statistical factors, the final (weakest) statistical factors, or any set of factors between.[5] The common volatility series extracted from these alternative sets of factors are very highly correlated. This indicates that a single common volatility feature exists throughout the spectrum of return variation, including strong factors, weak factors, and idiosyncratic return components.

If common factor volatility captures long-run equilibrium relations among factor volatility series, deviations from the long-run relation should forecast future innovations in factor volatility. We test implications associated with this hypothesis by estimating a simple error correction model. We first project factor volatility onto CFV for our broad set of factors, industry portfolios, and anomalies. With R^2s of 40%-70%, these regressions reiterate the pervasiveness of CFV. In a second step we investigate whether residuals from projections of volatility series on CFV, which can be interpreted as deviations from a long-run equilibrium relation with CFV, Granger-cause the corresponding volatility series. We find that they do. Deviations from the equilibrium relation predict reversals in future volatilities for the market as well as for most of the large set of equity factors we consider, even after controlling for lags of each series. The predictive power of equilibrium deviations with respect to CFV is economically, as well as statistically, significant, and CFV improves factor volatility forecasts relative to benchmarks out-of-sample as well as in-sample. To the extent that CFV reflects a ubiquitous equity risk feature, it is also of interest to test for a positive risk-return relation using CFV as the risk proxy as opposed to conventional risk proxies based on the volatility of a broad market index. We present evidence that CFV positively and significantly forecasts excess stock market returns over a variety of horizons. The predictive power of CFV exceeds that of a traditional stock return volatility measure, especially at shorter horizons.

The final portion of the paper explores the potential sources of commonality in factor volatility. Perhaps the most obvious candidate explanation is that common variation in fundamental uncertainty drives associated commonality in the various components of stock

[5] We also construct random long short portfolios by allocating stocks randomly into terciles every year and computing returns for the portfolio long the top and short the bottom tercile. We show that the volatility of these 10 portfolios also shares a common component, which is highly correlated with CFV.

return volatility. Although our measure of common volatility is persistent and cyclical, similar to most prominent risk measures in the literature, we show that common volatility only weakly correlates with a wide range of measures of aggregate economic uncertainty in levels, first differences, and long-run components.[6] Consequently, much of the variation in the common risk concept captured by CFV appears specific to the space of equity markets.

We conduct an additional test concerning the relation between fundamental uncertainty and common equity volatility that focuses on narrow time windows around firm earnings announcements. Earnings announcements should coincide with 'lumpy' arrivals of news concerning fundamentals and therefore we analyze the relation between the common component in the volatility of fundamental news and the common component in stock return variation associated with this news. Specifically, we construct measures of aggregate variation in the volatility of firms' earnings announcement surprises, as well as measures of aggregate variation in the volatility of stock returns during a narrow window around earnings announcements. There is a strong common component in both the earnings-based volatility measure as well as the return reaction-based volatility measure. We find evidence of a relatively strong (positive) relation between the common component of the volatility of market reactions to earnings news, but *not* with respect to the common component of the volatility of earnings surprises themselves. This indicates that variation in CFV relates less to common variation in the magnitude of shocks to fundamentals, and more to common variation in the virulence of market reactions to news. It is possible that a form of aggregate time-varying leverage explains this pattern of results. Therefore, in a second set of tests, we examine the relation between CFV and measures of financial and operating leverage, as well as proxies for the aggregate level of growth options. Results indicate that these measures only weakly relate to CFV. Our return-forecasting results lend some credence to an alternative hypothesis that CFV reflects discount rate variation. However, whether CFV ultimately relates more closely to discount rate variation or time-varying, behaviorally-driven 'excess volatility' in the spirit

[6] These include the economic policy measure of Baker, Bloom, and Davis (2016), the macroeconomic uncertainty measure of Jurado, Ludvigson, and Ng (2015), and a measure of real economic uncertainty developed by Ludvigson et al (2018).

of Shiller (1981) remains unresolved.

Relatively few papers focus on characterizing time-varying volatility for factors other than the market. Connor, Korajczyk, and Linton (2006) estimate a dynamic approximate factor model for US equity returns that includes a common feature in both factor variation and asset-specific variation. They model the conditional volatility associated with a particular linear combination of the latent factors using a parametric approach that includes both a secular trend and cyclical dynamics.[7] A stream of research finds a common factor in the *idiosyncratic* volatilities of individual firms (e.g., Campbell, Lettau, Malkiel, and Xu (2001), Connor et al. (2006), Duarte, Kamara, Siegel, and Sun (2014), Herskovic et al. (2016)), in contrast to our focus on the volatility of systematic factor returns. Christoffersen, Lunde, and Olesen (2019) identify common variation in the volatility of commodities post-financialization. This common commodity volatility feature correlates positively with equity market volatility. Christoffersen and Langlois (2013) find evidence of asymmetric tail dependence across the Fama-French and Carhart factors. Campbell et al. (2001) report that market, average industry, and average individual firm volatilities are correlated, but they do not explicitly analyze the volatility of other systematic equity factors. Barroso and Maio (2019) model time-variation in factor volatility in the context of analyzing the risk-return trade-off for prominent characteristics-based factors. Engle and Marcucci (2006) develop the pure variance common feature model and apply it to Dow Jones stocks, concluding that a few variance features capture commonalities in volatility for these stocks.

[7]In contrast to the semi-parametric approach in Connor et al. (2006), we adopt a 'realized volatility' approach to nonparametrically measure factor variances. Connor et al. (2006) focus on (latent) approximate factor models, whereas we extract measures of common volatility for an array of alternative classes of factor models and show that these produce virtually the same common volatility series. Connor et al. (2006) restrict attention to the aggregate variance of the factors they extract; they do not investigate whether the variances of individual factors are correlated with each other. We complement their results by showing that there is a single common factor that explains a large proportion of the variance of a range of statistical and characteristic-based factors.

1. Background and Motivation

Consider a standard factor model for excess returns on N stocks:

$$R_t = \Lambda F_t + e_t, \quad t = 1, \ldots, T, \tag{1}$$

where R_t denotes an $N \times 1$ vector of excess stock returns, F_t denotes an $K \times 1$ vector of factor realizations with $K << N$, Λ denotes an $N \times K$ matrix of factor loadings, and $\text{Cov}(F_t, e_t) = 0$. In matrix form, the model of Eq. (1) can be written

$$\underbrace{R}_{T \times N} = \underbrace{F}_{T \times K} \underbrace{\Lambda'}_{K \times N} + \underbrace{e}_{T \times N}. \tag{2}$$

At this stage, we leave the identity of the factors F_t unspecified. In particular, the factors might be observed or latent. Below we discuss alternative approaches to obtaining factors.[8]

Under the assumption that $E_{t-1}(F_t e_t') = 0$ for all t, we can write the conditional covariance of excess returns as:

$$\Omega_{R,t} = \Lambda \, \Omega_{F,t} \, \Lambda' + \Omega_{e,t}, \tag{3}$$

where $\Omega_{R,t}$ denotes the $N \times N$ conditional covariance matrix of excess returns, $\Omega_{F,t}$ represents the $K \times K$ factor conditional covariance matrix, and $\Omega_{e,t}$ is the $N \times N$ conditional covariances matrix for idiosyncratic errors. Equation (3) is a conditional version of the usual expression for the unconditional variance of returns under a factor model.

Our paper can be seen in the context of the large literature on multivariate volatility models (see Bauwens, Laurent, and Rombouts, 2006, for a survey). Multivariate volatility models such as the VECH model in Bollerslev, Engle, and Wooldridge (1988) specify dynam-

[8] When the factors are latent, they are identified only up to arbitrary normalizations. A common normalization imposes $(1/N)\Lambda'\Lambda = I_K$ and Ω_F is diagonal. Connor and Korajczyk (1986, 1988) develop an approach to consistently estimate latent factors when $N \to \infty$. Subsequent papers extend the approach to settings with N and $T \to \infty$, whilst also permitting heteroskedasticity and limited dependence in the cross-section and time series (e.g., Bai (2003)).

ics for each element of the covariance matrix of returns, $\Omega_{R,t}$. Factor volatility models, such as the factor ARCH model of Engle, Ng, and Rothschild (1990) or the heteroskedastic latent factor model of Diebold and Nerlove (1989), reduce the number of parameters to be estimated by exploiting the factor structure in returns as in Eq. (3). They obtain the dynamics of $\Omega_{R,t}$ by modelling the dynamics of the elements of $\Omega_{F,t}$. These models link commonality in variances across assets to common factor exposures in returns. They typically do not require/model commonality in the variance of the factors themselves. In contrast, the focus of our paper pertains to the extent to which the volatility dynamics of $\Omega_{F,t}$ *itself* contain a common component.[9]

The conditional covariance matrix of factors can be decomposed as $\Omega_{F,t} = D_{F,t}\mathrm{P}_{F,t}D_{F,t}$, where $D_{F,t}$ is a $K \times K$ diagonal matrix of conditional factor standard deviations and $\mathrm{P}_{F,t}$ is the $K \times K$ conditional factor correlation matrix. Letting $h_t = diag(D^2_{F,t})$ denote the $K \times 1$ vector of conditional factor volatilities, where $diag(X)$ extracts the vector of diagonal elements from matrix X, we hypothesize that equity factors follow a version of the *pure variance common feature* (PVCF) model introduced by Engle and Marcucci (2006):

$$h_t = \Gamma\, CFV_t + diag(\tilde{\Sigma}_t), \tag{4}$$

where CFV_t denotes an $M \times 1$ vector of common components of factor volatility such that $M < K$, Γ denotes a $K \times M$ matrix of loadings on the common component CFV_t, and $diag(\tilde{\Sigma}_t)$ indicates a $K \times 1$ vector describing 'idiosyncratic' conditional variances for the set of factors.[10] In this setup, assets can exhibit common dynamics in their variances even if they do not share exposure to the same (orthogonal) factors in returns, because the variances of the factors contain a common component. The PVCF model of Equation (4) posits that conditional variances depend on a small number of variance factors without requiring that covariances/correlations depend on these same factors.[11] We primarily focus on the case of

[9]This is trivially true in a single factor model ($K = 1$). We focus on multi-factor models in this paper, noting that such models are extremely common both in the literature and in practice.

[10]Latent factor models require an additional normalization assumption. See, e.g., Connor et al. (2006).

[11]Similar to cointegration (Engle and Granger (1987)), the PVCF model represents a special case of a

a single common volatility factor ($M = 1$), in which case CFV_t is a scalar time series and Γ is a $K \times 1$ vector of corresponding loadings.

Under the PVCF model, there exists a linear combination of the factor volatility series that lacks the feature, i.e., that exhibits purely idiosyncratic variation. However, this does *not* imply that there exists a linear combination of the traded factors – a feasible portfolio fully invested in these factors or stocks underlying the factors – that lacks exposure to the common volatility feature. Indeed, a standard minimum variance argument implies that this will not be the case and therefore common factor volatility represents a fundamental form of equity risk.

Theory provides limited guidance regarding the relative importance of the common volatility feature(s) CFV_t versus the idiosyncratic component of time-varying factor volatility in Equation (4). In particular, nothing in the underlying framework of an approximate statistical factor model, along with the corresponding conditions necessary to deliver the arbitrage pricing theory (APT) of Ross (1976) and Chamberlain and Rothschild (1983) necessarily implies the existence of a strong common component in factor volatility. Indeed, it is possible to posit an approximate statistical factor model for which the standard APT result holds, and such that factor volatilities vary over time but in a completely idiosyncratic manner. On the other hand, consumption-based asset pricing models typically imply a stochastic discount factor that involves a small number of macroeconomic risks, e.g., shocks to current and long run future consumption growth in prominent long run risk model of Bansal and Yaron (2004). To the extent that a set of traded return factors F_t jointly capture these few underlying macroeconomic risks, it seems plausible to expect associated comovement in factor volatility. Ultimately, therefore, the extent of common variation in factor volatility is an empirical question and represents the primary aim of our study.[12]

common feature among economic time series. Engle and Kozicki (1993) formally define a common feature as arising whenever a set of time series exhibit the feature, but a linear combination of the series fails to exhibit the feature.

[12]It is also possible that, in addition to a common component in factor volatility, there also exists commonality in time-variation among factor correlations contained in $\mathrm{P}_{F,t}$. In addition to measuring the common component in factor volatility CFV_t, we also explore the extent to which there is a common component in time-varying factor correlations.

It is necessary to specify factors in order to empirically measure commonality in time-varying volatility. One possible approach is to estimate a latent factor model using, for example, principal components in the spirit of Connor and Korajczyk (1986, 1988), etc. However, there are many other approaches to specifying factors in the literature, including characteristics-based factors, factors based on portfolios intended to mimic underlying macroeconomic risks, and so forth. Rather than champion a particular approach, we consider a wide range of factor models. Ultimately, we demonstrate that essentially the same common volatility component emerges irrespective of the particular factor model specification.

Many theoretically-motivated factor models include a market factor defined as the excess return on a broad value-weighted portfolio. In addition, empirical applications of statistical approaches to factor models typically identify a first factor that is highly correlated with the excess return on a broad value-weighted portfolio. Given that market volatility dynamics are relatively well-understood, we focus on assessing the degree and nature of commonality in the volatility of factors *beyond* the market factor. We therefore extract measures of a common component from sets of factors that exclude the market factor. We then compare the resulting common factor volatility series CFV_t with both market volatility and a measure of the common idiosyncratic volatility CIV_t. Ultimately, we show that there is a substantial degree of comovement among these three series.

2. Commonality in Factor Volatility

2.1. Measuring Common Factor Volatility

We measure factor volatility using a nonparametric approach motivated by the fact that high frequency return observations enable the precise measurement of volatility (see, e.g., Andersen, Bollerslev, Diebold, and Labys (2003)). Let t index periods at a frequency over which we aim to measure factor volatility, and $j \in [1, 2, \ldots J_t]$ index higher frequency observations of factor returns within the t-th period. In most applications t will index monthly observations with j indexing trading days within a month. In certain cases t will instead be annual and j will index months within the year, owing to data limitations. Given an arbitrary set of K traded factor returns, $F_{j,t}$ denotes the $K \times 1$ vector of factor returns on the j-th day of period t and $F_{k,j,t}$ denotes the k-th factor return on this day. We measure raw factor variances as the so-called 'realized variance' computed as the sum of squared daily factor returns during the corresponding period:

$$\hat{\sigma}^2_{F,k,t} = \sum_{j=1}^{J} F^2_{k,j,t}. \tag{5}$$

Raw factor volatility $\hat{\sigma}_{F,k,t}$ equals the square root of the variance defined in Eq. (5).

In addition to analyzing raw factor volatility series, we also measure the volatility of factor returns that are orthogonalized with respect to the market factor. Let $MKT_{j,t}$ denote the excess return on a market proxy. Define a market-adjusted factor return as:

$$\epsilon_{k,j,t} \equiv F_{k,j,t} - \hat{\beta}_{k,j,t} MKT_{j,t} \tag{6}$$

where $\hat{\beta}_{k,j,t}$ denotes an estimate of the (potentially time-varying) market beta. We then compute market-adjusted factor variances as:

$$\hat{\sigma}^2_{F,k,t}(\text{market adjusted}) = \sum_{j=1}^{J} e^2_{k,j,t}, \tag{7}$$

with associated factor volatilities equal to the square root of Eq. (7).

We base market beta estimates for factors on an OLS regression of returns for the k-th factor on market excess returns estimated over a specified window. There is a form of bias-variance trade-off with respect to the window choice. Betas likely change over time, and a narrower window reduces bias associated with such time-variation. On the other hand, a tighter window reduces the precision of estimates. For most reported results, we adjust factor returns using a beta estimate computed from an OLS regression of daily factor returns on daily data over the corresponding calendar year. Section 3 considers beta estimation issues and robustness of results to alternative windows.

We consider two approaches for measuring the common component in factor volatility. The first defines the common component as the cross-sectional average of the volatilities for the corresponding factor set. The second approach defines the common component as the first principal component extracted from the standardized factor volatility series. The two approaches generally produce highly correlated common component series.

2.2. Factor Sets

This section describes the various sets of factors we analyze. Our primary proxy for the market factor MKT is the market excess return factor from the Fama-French research factor library. The underlying market return measure includes CRSP firms listed on NYSE, Nasdaq, or AMEX with share codes of 10 or 11. When a risk-free rate is required, we use the one-month Treasury bill rate from Ibbotson Associates.

We now describe the particular factor sets considered in our analysis. Data sources and additional details for each set of factors appear in the Supplementary Appendix.

1. **Characteristics-based factors:** A large number of factors have been proposed and we analyze only a subset of prominent factors from the literature. These include the size (SMB) and value (HML) factors from the prominent three-factor model of Fama and French (1993), and the investment (RMW), and profitability (CMA) factors recently proposed by Fama and French (2015) in an extended model. We also analyze several more recently proposed characteristics-based factors. These include two mispricing measures proposed by Stambaugh and Yuan (2016), one pertaining to management decisions (MGMT) and the other related to performance (PRF), the 'betting against beta' (BAB) factor proposed by Frazzini and Pedersen (2014) and the 'quality minus junk' (QMJ) factor constructed by Asness, Frazzini, and Pedersen (2014).[13]

[13] Our empirical approach relies on the availability of relatively high-frequency factor returns (daily) to construct measures of return variation and co-variation. This requirement excludes some prominent factors in the literature. For example, we do not analyze the traded liquidity factor of Pástor and Stambaugh (2003) because the factor is only available at a monthly frequency. In robustness checks, we substitute alternative investment and profitability factors from Hou, Xue, and Zhang (2015) and obtain similar results. We thank Ken French, Robert Stambaugh, Lu Zhang, and AQR for making factor return data available.

2. **Anomaly long-short portfolios:** Although popular in practice, standard characteristics-based factor models appear to be imperfect. Indeed, Kozak, Nagel, and Santosh (forthcoming) argue that characteristics-sparse models cannot adequately summarize the cross-section of expected stock returns. Consequently, we also analyze returns for portfolios constructed based on characteristics associated with a large number of stock return anomalies identified in the literature. We consider several alternative sets of anomaly portfolios. A first approach follows Green, Hand, and Zhang (2017), who sort firms into deciles each month based on each individual characteristic and form long-short portfolios using extreme deciles. We form and update portfolios on a monthly basis as in Green et al. (2017) (dubbed GHZ anomalies), but construct *daily* returns for these portfolios in order to form volatility measures. A second set of returns consist of the daily anomaly returns constructed and analyzed by Kozak et al. (forthcoming), referenced as *KNS Anomalies*. In both cases, we omit some anomalies from the original sets due to limited historical coverage. The resulting set of 62 GHZ anomaly portfolios covers the period 1964.7–2018.12 and the set of 41 KNS anomaly returns covers 1963.11–2017.12.

3. **Industry Portfolios:** This set of factors is based on industry returns.[14] We base the industry factors based on Fama-French value-weighted and equal-weighted 12, 17, and 30 industry portfolios. We designate variations as, e.g., 'INDU-12VW.' In contrast to the characteristics-based factors and anomaly returns discussed previously, the industry factors are long-only portfolios.

4. **Macroeconomic factors:** Following Chen et al. (1986), we consider a set of macroeconomic risk measures. These include the growth rate in industrial production, unexpected inflation, the change in expected inflation, the term premium, the default

[14]The returns on such portfolios constitute factor returns under the simple form of fundamental factor model in which (observed) factor loadings correspond to a set of K mutually exclusive dummy variables representing the industry membership for each firm. This is sometimes termed the 'BARRA-type industry factor model,' as it was developed by Bar Rosenberg, founder of BARRA Inc. (see, e.g., Grinold and Kahn (2000) and Conner et al (2010)).

spread, a measure of real consumption growth, and the log change in oil prices. We follow Cooper and Priestley (2011) and create mimicking portfolios for all factors. Factor mimicking portfolios are created based on projections of monthly macroeconomic risk series on monthly returns for 100 portfolios. Daily mimicking portfolio returns are computed based on underlying monthly tracking portfolios to measure volatility. The Supplementary Appendix provides additional details.

5. **Statistical Factors:** We construct statistical factors based on the principal components extracted from a large set of daily portfolio returns. We utilize portfolios returns, rather than daily returns for individual stocks in order to mitigate the potential influence of microstructure effects and extreme daily returns for individual stocks on the principal components. To construct factors, we follow the 'Risk Premium PCA' (RP-PCA) approach recently proposed by Lettau and Pelger (forthcoming). This approach extends the standard (asymptotic) Principal Component Analysis (PCA) (Connor and Korajczyk (1986), Connor and Korajczyk (1988)) method by incorporating a penalty term that reflects the magnitude of cross-sectional pricing errors. RP-PCA encourages the identification of factors with high Sharpe ratios that fit the cross-section of expected returns as well as explain time-series comovements. The Appendix provides formal discussion of the approach and describes details regarding our implementation. We consider several sets of underlying portfolio returns as the basis for statistical factors. The first set consists of 105 portfolios including daily returns for the Fama-French 25 size- and book-to-market-sorted portfolios, 25 size- and momentum-sorted portfolios, 25 size- and (long term) reversal-sorted portfolios, and 30 Fama-French value-weighted industry-sorted portfolios ('FF+IND'). The second set consists of the GHZ anomaly portfolios and the 30 value-weighted industry portfolios ('GHZ+IND'). The final set consists of the KNS anomaly portfolios discussed previously along with the 30 value-weighted industry portfolios ('KNS+IND').

2.3. Commonality in Factor Volatility: Evidence

Figure 1 provides visual evidence of a common component in factor volatilities. The figure shows quarterly volatility series for characteristics-based factors. Panel A presents volatility series based on raw factor returns. Panel B shows volatility series based on market-adjusted factor returns. Individual factor volatility series appear as labeled, colored dashed-dot lines. Panel A of Figure 1 illustrates that the raw factor volatility series exhibit a striking degree of co-movement. Moreover, the common variation in characteristics-based factor volatility appears to co-move positively with market volatility, shown as a blue solid line for reference.

It is possible that the common variation in factor return volatility series and the co-movement of this common variation with market volatility is simply due to the fact that characteristics-based factor returns are not market neutral. For example, it is possible that average market betas differ for stocks in the long versus short legs in these portfolios. Moreover, even if factor betas are unconditionally small, they might exhibit substantial time-variation. Panel B addresses this concern by showing quarterly volatility series for *market-adjusted* factor return series with respect to market return variation. Market-adjusted factor return volatility series continue to exhibit a strong common component that is strongly positively correlated with market volatility and negatively correlated with economic conditions.[15] To more clearly indicate the common source of variation, Panel B depicts a measure of common variation in (market-adjusted) factor volatility series, constructed as the equal-weighted average of the factor residual volatilities. The dynamics of the common component in factor volatility continue to be similar to those of market volatility. At the same time, there is visual evidence that the common volatility component differs in some respects from market volatility. For example, the behavior of the common volatility series in the late 1990s and early 2000s differs somewhat from market volatility. Later we explore in further detail the relation between market volatility and the common component of factor volatility.

Figure 2 provides plots of market-adjusted volatility series for two other sets of factors:

[15] Moreira and Muir (2017) note that characteristics-based factor volatilities co-move with market volatility (see their Figure 1), but do not explicitly analyze whether this is simply attributable to direct market exposure.

Panel A shows volatility series for market-adjusted returns on 12 value-weighted Fama-French industry portfolios. Panel B shows volatility series for market-adjusted returns on 12 randomly selected anomaly long-short portfolios from the KNS anomaly set. Both panels depict the equal-weighted average of the various industry or anomaly volatility series as a blue solid line. The main takeaway from Figure 2 is that the alternative industry and anomaly return sets produce qualitatively similar results to those for characteristics-based factors. In particular, the appears to be a strong common component in volatility for market-adjusted returns for both industry and anomaly portfolios. Both industry and anomaly return volatility series tend to peak in economic recessions and/or financial crises and are positively correlated with market volatility. The common volatility component inherits these features, and therefore is both countercyclical and positively correlated with market volatility. It is notable that the dynamics of the common volatility component constructed from industry and anomaly returns are both very similar to each other as well as very similar to the common volatility component constructed from characteristics-based factors in Panel B of Figure 1.

Table 1 presents statistics summarizing the evidence for a strong common component in factor volatility. Given a particular set of factor returns, we first compute market-adjusted daily returns and corresponding realized factor volatility measures at the quarterly frequency. We define a measure of common factor volatility (CFV) as the first principal component extracted from the set of factor volatility series,[16] The first column describes the set of factor returns analyzed. The second column indicates the sample period over which factor returns are observed. The third column of Table 1 (% Expl.) shows the percent of total factor volatility variance explained by the CFV series. The next two columns display the sample correlation between CFV and two reference series: market volatility and the ADS measure of economic conditions. Market volatility is computed as the sum of squared daily returns on the market factor downloaded from Ken French's website. The final three columns of Table 1 summarize the market exposure of the underlying factor returns. The statistic $\bar{\hat{\beta}}$

[16]The PC-based CFV series is typically highly correlated an alternative average volatility measure for most factor sets. However, in some instances, such as the CRR macroeconomic factors, the level of volatility can differ substantially across factors. For this reason ,we adopt the PCA common volatility measure.

shows the pooled (across portfolios and years) market beta estimates from the annual market model regressions. $\overline{SE(\hat{\beta})}$ is the average standard error for the beta estimates. Finally, $\bar{\hat{\beta}}$ equals the pooled (across portfolios and years) R^2-statistic for the market model regressions.

Panel A provides results for various subsets of characteristics-based factors. The CFV series explains a large proportion of variation in the volatility of factor returns (70–80% depending on the factor set). CFV is strongly positively correlated with market volatility (typical correlations are around 0.8) and consistently negative correlated with the ADS measure, confirming a countercyclical relation. Pooled average market beta estimates are generally close to zero. The pooled average R^2-value is consistently in the 20-25% range for the various factor portfolios. This indicates that, although the long-short factor portfolios are market neutral on average, they do exhibit market exposure via time-variation of market betas around zero. Finally, annual market betas are estimated reasonably precisely, with typical standard errors of approximately 0.03–0.04.

Panel B presents results for anomaly long-short portfolios. These are also qualitatively similar to earlier results for characteristics-based factors. There is again strong evidence of a factor structure in residual volatility: the percentage of anomaly portfolio residual volatility explained by the common component is approximately 82% for the more limited anomaly set and roughly 75% for the full set of anomalies. The CFV series is strongly positively correlated with market volatility negative correlated with economic conditions.

Panels C, D, and E present results for industry, macroeconomic, and statistical factors, respectively. Results are qualitatively similar to those for characteristics-based factors. The fraction of variance explained by the common volatility factor remains economically large for each of these sets of alternative factors. The CFV series associated with these factor sets are again positively and strongly correlated with market volatility and negatively correlated with economic conditions. Unsurprisingly, the average level of market exposure is greater for the long-only industry portfolios relative to long-short factors or statistical factors.

Each of the alternative CFV measures extracted from various sets of factor and anomaly returns are highly positively correlated with market volatility. This suggests that they are also likely to be highly correlated with each other. Figure 3 confirms this conjecture. The figure plots five alternative CFV measures extracted from different sets of portfolio returns. The series are standardized to facilitate comparison and plotted over the overlapping sample period of 1965–2018. The measures are extremely similar. Pairwise correlations (not explicitly reported) are generally in excess of 0.9. Below, we will often speak of *the* CFV series. As the picture indicates, the particular choice of variation on the measure is relatively unimportant, and results are highly robust to using other variations.

2.4. Commonality in Correlations

The discussion to this point focuses on commonality in the dynamics of volatility for factor and anomaly portfolios. We now briefly consider the behavior of *correlations* for such portfolios. The key questions we address concern whether there a strong commonality also exists among these correlations, and what is the relation between the dynamic evolution of correlations versus the common volatility series previously described.

Table 2 provides summary statistics regarding quarterly correlation measures for raw and market-adjusted factor and anomaly portfolios. For each portfolio within a specified set, we compute time series of quarterly realized correlations as the sample pairwise correlations of daily returns within the corresponding quarter. As an example, if the factor set consists of five factors, we obtain 10 quarterly time series, each reflecting the dynamics of a particular pairwise correlation among these factors. We then compute a simple measure of the common component of the correlations as the average of the pairwise correlations within each quarter, denoted $\overline{\rho}_t$. The table shows the percent of total variation captured by this common component measure ('% Expl.'), as well as the annualized time series standard deviation of the common correlation measure, denoted $\sigma(\overline{\rho})$. To shed light on whether the dynamics of the common correlation component are similar to that of market volatility or economic conditions, we report the time series correlations between the common correlation measure and market volatility and between the common correlation series and the ADS index. The right-hand side of Table 2 provides similar statistics for correlations constructed using market-adjusted factor returns. This helps convey to what extent common correlation arises due to common market exposure. The common correlation measures explain much less of the total variation for most portfolio sets relative to the common volatility series described in Table 1. For raw factor correlations, the percent explained is between 10–30% with the exception of the long-only industry portfolios which are subject to common market exposure. Upon examining correlations constructed from market-adjusted returns (right-hand side of Table 2), the fraction of variation explained by the first principal component of the correlation series is always under 30%. Correlations with both market volatility and economic conditions are mixed in sign and generally economically weak. Figure 4 illustrates correlation dynamics for the factors SMB, HML, RMW, CMA, and UMD. These five factors give rise to a set of 10 pairwise correlations. Panel A of Figure 4 plots quarterly realized correlations for raw factor returns. To facilitate interpretation, the plot shows correlations as differences from their full time series average value. The 10 correlation pairs are plotted as dashed-dot lines without labeling to avoid clutter. The thick solid line depicts the average of the 10 correlation pairs. The correlation pairs exhibit significant variation over horizons of several years. However, there does not seem to be a strong degree of coherence with respect to this variation, as the average correlation is considerably less volatile. Furthermore, common correlation does not appear to closely relate to stock market volatility or the CFV series. Panel B illustrates that the correlations of 'market-adjusted' characteristics-based factor returns exhibit similar properties. The common component fluctuates relatively little, save a notable increase in the early 2000s. The bottom line from this analysis is that, while factor return correlations exhibit time series variation, there does not appear to be a dominant common factor, and the common component of variation does not consistently closely relate to market volatility, CFV, or economic conditions.

3. Common Volatility: Extensions and Robustness

This section extends results concerning common volatility in several directions. First, we address critiques related to potential spurious sources for apparent common factor volatility and present results for associated robustness checks. Second we consider the relation between the common factor volatility of central interest in this paper and previously documented common variation in firm-specific return components.

3.1. Spurious Common Factor Volatility?

A first potential concern is that measurement error with respect to market beta estimates might induce an apparent common volatility component. To appreciate the issue, assume momentarily that the true beta for a particular factor equals one and that an econometrician follows our procedure to obtain a measure of market-adjusted volatility. Suppose that she obtains a beta estimate of zero, rather than one, and computes market-adjusted returns and associated volatility measures using this incorrect beta estimate. Mechanically, the (estimated) residuals used to construct volatility measures contain a component equal to daily market returns. If similarly egregious estimation errors occur across multiple portfolios, it will appear as though there exists a common volatility component that is highly correlated with market volatility, even if no such common component exists. The Appendix provides additional analysis regarding the implications of beta estimation error. We consider a setting in which daily asset or portfolio returns follow a standard single factor (market) model with potentially time-varying market betas and i.i.d. idiosyncratic returns. A spurious common variance component mechanically related to market variance occurs when *either* betas are constant within period but the periodic beta is estimated with error, *or* the periodic beta is estimated accurately, but there is daily variation around this average within the period.

We present several robustness checks indicating that beta measurement error is unlikely to explain the common factor volatility we observe. Panel A of Table 3 shows summary statistics for CFV series extracted from factor and anomaly returns when the market model regression used to obtain residuals is estimated *monthly*, as opposed to annually as in Table 1. Monthly betas better adapt to potential within-year variation in betas, which constitute one potential source of spurious common volatility. To conserve space, we report results only for selected sets of portfolios. The results for these portfolios are extremely similar to those for the corresponding portfolio sets in Table 1 based on annual regressions. As a second robustness check, we analyze several sets of portfolios where there exists strong prior knowledge concerning true market betas. These include characteristics-based factors, long-short anomaly returns, and statistical factors extracted from returns. In the first two cases, the long-short nature of the portfolio is likely to produce returns with relatively low market exposure. For the statistical factor return set, the first factor is likely to be highly correlated with the market factor (in the data, the correlation is over 0.9). We therefore jettison this factor and examine the next ten factors, which should be close to uncorrelated with market returns. Panel B in Table 1 shows results for these factor sets based on raw returns, i.e., imposing that $\hat{\beta} = 0$, in contrast to the results for market-adjusted returns reported in Table 1. The results are extremely similar. Figures presented in the Supplementary Appendix show that the common volatility series for these portfolios are nearly identical to those based on market-adjusted returns. This would be unlikely to occur if beta estimation error was the sole source of apparent common volatility.[17]

A second potential concern is that commonality in time-varying microstructure noise might masquerade as common factor volatility. The 'realized variance' proxies we compute for factor volatility are based on daily returns for portfolios that potentially include relatively

[17] As an additional test, we form long-short portfolios consisting of *randomly selected* firms from the same universe of stocks that comprise the basis for our anomaly portfolios. At the end of each calendar year, we generate a set of 10 randomly generated artificial 'characteristics' for each eligible stock in the sample at that time. We then form random long-short portfolios by sorting stocks into terciles based on each of these randomly generated characteristic values and forming associated long-short portfolio based on the extreme terciles. We compute value-weighted daily returns for these portfolios over the ensuing calendar year, and repeat this process for each calendar year. Given the construction of these portfolios, the market beta of the long-short portfolios should be close to zero at all times. If market beta estimation error is the driving force behind the common volatility component, we should observe a strong common component when estimating betas for these portfolios, but not when we impose the virtually correct restriction that $\beta = 0$. Instead, we find a very similar common component under both approaches.

illiquid stocks. We consider two associated robustness checks. First, we construct alternative daily factor returns for the Fama-French five factors based on the mid-point between the daily bid and ask closing prices recorded by CRSP. These returns cover the period 1993–2018. Results in Panel C for the four non-market factors constructed from the bid and ask prices are very similar to results constructed from CRSP closing prices.

Monthly returns should be much less affected by microstructure noise relative to daily returns. Consequently, we construct alternative set factor volatility series based on squared *monthly* returns, rather than daily returns. Monthly factor returns are adjusted for market exposure by running market model regressions using a rolling window of 60 months (five years) of data. We then construct factor volatility series as the sum of the past 12 squared monthly residuals and extract a common factor volatility series using the same approach as in our main analysis. Table 3 shows results. Results are qualitatively similar to those reported in Table 1, indicating that microstructure noise is unlikely to be the source of the common factor volatility we document.

We conducted yet further robustness checks concerning the presence of a strong common component among factor volatility series. Common volatility measures extracted from monthly realized volatility series (rather than quarterly) exhibit similar properties to those reported in Table 1. We also consider alternative ways to construct statistical factors, such as using the traditional PCA approach instead of RP-PCA, or examining a longer time series of statistical factor returns available since 1927. All variations produce similar results.

3.2. Common Factor Volatility versus Common Idiosyncratic Volatility

Several previous studies conclude that there is common variation in the volatility of idiosyncratic returns. Apparent common *factor* volatility might arise as an artifact of previously documented commonality in idiosyncratic volatility. In particular, value-weighted factor portfolios can contain 'granular measurement errors' driven by failure of the law of large numbers associated with the fat-tailed nature of the size distribution of public firms.[18] To address this potential concern, Panel D of Table 3 shows results for equal-weighted, as opposed to value-weighted, industry returns. These portfolios should be relatively insulated from granular measurement errors and virtually free of truly idiosyncratic return components. We continue to find strong evidence of common factor volatility.

We also consider the reverse possibility: does apparent common idiosyncratic volatility arise solely due to exposure to so-called 'weak factors' in market-adjusted (or Fama-French factor-adjusted) return residuals that are treated as idiosyncratic in some previous studies? To investigate this question, we identify common idiosyncratic volatility via an alternative approach: we test for commonality in volatility in idiosyncratic portions of the spectrum of return variation produced by a statistical factor decomposition. In particular, instead of analyzing the leading factors extracted by RP-PCA, we instead analyze alternative sets of statistically extracted factors that explain increasingly *less* variation in returns. We continue to strong evidence of a common volatility component across these alternative factor sets. Moreover, the common component extracted from the idiosyncratic spectrum of the factor decomposition is very similar to that extracted from the leading 'strong' and 'semi-strong' factors reported in earlier results. Figure 5 plots the standardized common volatility series extracted from four alliterative sets of statistical factor constructed from an underlying set of 87 GHZ anomaly and industry portfolio returns. Similar results obtain for statistical factors constructed using alternative underlying stocks or portfolios. We draw two conclusions from this analysis: 1) there does indeed appear to be commonality in idiosyncratic volatility; and 2) the common component in factor volatility and idiosyncratic volatility are very highly correlated. The latter point is important, as it implies that, effectively, a single common volatility feature pervades the entire spectrum of equity returns.

Figure 6 provides additional visual evidence concerning the relations among CFV, market volatility, and CIV.[19] Panel A of Figure 6 plots annual time series for each volatility measure.

[18] Byun and Schmidt (2018) find that asset pricing tests are sensitive to the presence of such granular measurement errors.

[19] We construct the common idiosyncratic volatility (CIV) measure of Herskovic et al. (2016) as the equal-weighted average across stocks of monthly idiosyncratic variances. These variances are computed as the sum

To facilitate comparison among series, we plot the standardized value of the natural logarithm of each volatility series. There is strong affinity among CFV, market volatility, and CIV, consistent with the high correlations reported in Table 9. While both (standardized) CIV and MVOL occasionally deviate from CFV, these deviations are relatively short-lived. In other words, CFV appears to serve as a 'gravitational attraction point' for market volatility and CIV. Panel B plots the time series of deviations of market volatiity and CIV from CFV. This plot illustrates that both market volatility and CIV tend to 'wander' away from CFV, but these deviations do not appear to be transitory rather than permanent. To the extent that both series exhibit a long-run equilibrium relation with CFV, the speed of adjustment seems relatively slow, in the sense that deviations can persist for a number of years. One concrete example involves the behavior of market volatility relatively to CFV over the past several decades. Standardized market volatility is persistently above CFV during the financial crisis and its immediately aftermath, but more recently fell significantly below CFV.[20] The informal evidence in Figure 6 suggests that CFV can be interpreted as a type of "fundamental volatility" quantity, such that other equity volatility series, including factor volatilities and idiosyncratic volatility series, share a long-run relation with this fundamental series. Below we explore this theme further.

4. Implications of Common Factor Volatility

This section investigates the properties of common factor volatility. We first characterize the time series persistence of CFV and the extent to which CFV co-moves with a variety

of daily squared residuals from regression on the Fama-French three factor model. We make one substantive alteration to the Herskovic et al. (2016) measure: we exclude very small stocks, defined as those firms with market capitalization below the 20th percentile of NYSE-listed market capitalization at the end of the previous calendar year. We do this in order to increase the degree of comparability across time for the (unbalanced) panel of stocks that form the basis for the CIV time series measure. The Supplementary Appendix provides an explicit comparison of the alternative versions of the series. The differences are most pronounced during the 1990s, which coincides with the 'dot-com' era and an associated period of high IPO rate among relatively young, unprofitable firms.

[20]The Supplementary Appendix plots the relation between *long-run* components of the three volatility series extracted via a simple filtering procedure. This plot indicates that deviations of the (standardized) long-run components of market volatility and CIV from that of CFV also have a tendency to reverse.

of other volatility, uncertainty, and risk measures from the literature. We then examine the predictive power of CFV for other volatility series in the context of error correction models. Finally, we briefly explore the relation between CFV and the equity premium. It is necessary to settle on a particular measure of common factor volatility. Results that follow use as CFV the equal-weighted average of market-adjusted volatilities for 10 Fama-French equal-weighted portfolios. Because CFV series extracted from different factor sets are highly correlated, the specific choice has little effect on the main results that follow. The Supplementary Appendix provides explicits evidence concerning robustness to alternative choices.

4.1. Common Factor Volatility as a Predictor of Future Volatility

In Tables 5 and 6 we examine the relationship between monthly CFV and volatilities of the five Fama and French (2015) factors as well as UMD of Carhart (1997). If a long-run equilibrium relation exists between CFV and volatilities of traded assets, we expect deviations of such traded factors' volatilities from their equilibrium (with respect to CFV) to have predictive power for future volatility. We first estimate the following OLS model for alternative factor volatility series:

$$log(\sigma_{F_k,t}) = a + \beta \cdot log(CFV_t) + e_t, \tag{8}$$

where $\sigma_{F_k,t}$ denotes the volatility series for factor k and CFV_t is common factor volatility. The parameter β measures the loading strength of factor k volatility on the common volatility component, and the regression R^2-value reflects the proportion of variation in factor k volatility attributable to this common component. Panel A of Table 5 reports detailed results for the six factors (including the market factor). There is a strong linear relationship between the natural logarithms of factor volatility and CFV for all six factors. Estimates of the parameter β capturing the loading on the common component range from 0.54 to 0.78. The regression R^2-values range from 0.29 for market volatility to 0.61 for RMW volatility. This indicates that the common volatility component explains a significant share of time series variation in the factor volatility series and is consistent with earlier results regarding the economic significance of common factor volatility. The residuals from the regression model of Eq. (8) can be interpreted as transitory factor volatility deviations from a long-run equilibrium relation with CFV. In Panel B and C we report correlations of log volatilities and correlations of residuals from Equation 8. The pairwise correlations of the six factors in Panel B all exceed 0.60. Once we project off CFV, the pairwise residual correlations are all positive but relatively low in magnitude. Only the correlation between Mrktrf and SMB remains high. Most other pairwise correlations are reduced by approximately fifty percent or more. This reiterates the notion that a large part of commonality in these portfolios' volatility is a associated with a common factor volatility. The small magnitude of the correlations in Panel C also indicates that, at the monthly frequency, deviations from a linear functional relationship between CFV and each of the six factors are positively, but not strongly correlated.

Figure 7 summarizes results for regressions similar to those reported in Table 5 for a variety of alternative sets of factor and anomaly portfolios described in Tables 1, 2 and 4, *all of whose daily returns have been orthogonalized with respect to market returns.* The figure shows histograms summarizing empirical estimates of loadings parameters for these factor volatility series on CFV (β in Eq. (8)) and R^2-values for the loadings regressions. The histograms of loadings estimates in Panel A illustrate that virtually all factor and anomaly volatility series we consider load positively on CFV. The histograms in Panel B show that the vast majority of R^2-values for the loadings regressions cluster in the range of 0.4–0.6. Jointly, these results indicate that a wide range of diversified equity portfolios exhibit time-varying volatility that correlates significantly with CFV.

Table 6 examines the power of CFV for predicting *future* volatilities of the six factors. The presence of a long-run relation among factor volatility series suggests that deviations from long-run relations should predict future volatility trajectories via an error correction mechanism. For example, a large positive current residual implies that current factor volatility is substantially above its equilibrium with respect to CFV. We consequently expect lower volatility of that factor as the long-run equilibrium relation is restored. With this motivation in mind, we test whether a factor volatility's current deviation from its long-run relation with CFV (i.e., the residuals from the estimated linear function given in Equation 8) contains predictive power for future volatility, conditional on past factor volatility.

Each panel of Table 6 presents results for a single factor. We examine predictive regressions of the form

$$log(\sigma_{F_k,t+1}) = a + b'\mathbf{X}_t + \gamma'\mathbf{Z} + e_{t+1}, \tag{9}$$

where $\mathbf{X}$ denotes a d-dimension vector of predictors where $d \in \{1,2\}$ with elements of $\mathbf{X}$ coming from the set of random variables $\{log(\sigma_{F_k}), log(CFV), resid\}$. The vector $\mathbf{Z}$ includes additional lags of factor volatilities σ_{F_k}, from the previous 3 months and the previous 12 months. We control for these past lagged volatilities at differing horizons following the HAR model which Corsi (2009) shows is able to capture the long memory of equity volatility series. Specification (4) includes these additional lags, for all other specifications $\gamma = 0$. In Panel A, we see that CFV has substantial predictive power for future volatility of the market factor. Market factor volatility and CFV have R^2's of 0.44 and 0.26 respectively for predicting future market volatility at the 1 month horizon.

In specifications (3) and (4), we examine the predictive power of factor volatility residuals reflecting deviations from long-run relations with CFV. In all cases, we include lagged factor volatility as well as the lagged residual. The residual is highly significant with t-statistics exceeding 3 in magnitude. As conjectured, the slope coefficient estimate corresponding to the deviation from long-run equilibrium is negative, indicating that deviations from equilibrium tend to be reversed in future months. It is also notable that when we compare specification (1) with (3), we see that there is an economically significant increase in R^2 to be gained by including the residual as well as past factor volatility in Equation 9. In specification (4) we include additional lags of length 3 and 12 months of factor volatility. Although this slightly reduces the coefficient estimates, the slope estimate on the CFV residual term $resid_t$ remains negative and highly significant.

Panels B, C, D, E and F examine the remaining factors. In all cases, the results are very consistent. CFV is a strong predictor of each factor's subsequent volatility and residuals are strongly negatively related to subsequent factor volatility.[21] In unreported results (available upon request), we further show that the inclusion of lagged CFV in predictive regressions for each of the factors significantly improves out of sample forecasts of future volatility relative to the benchmark models used in Table 6. Figure 8 summarizes results for similar volatility forecasting regressions using a wide range of factors and anomaly portfolios beyond the the six factors analyzed in Table 6. The portfolios used in Figure 8, have all been orthogonalized with respect to the market factor in order to rule out the possibility that predictability is driven by portfolio returns loading on market returns. The vast majority of slope coefficient estimates on the deviation from long-run equilibrium with CFV are negative and significant in these forecasting regressions. This illustrates that the incremental forecasting power of deviations with respect to CFV is not specific to the Fama and French (2015) or Carhart factors and instead prevails across most factors and anomaly portfolios we analyze. Broadly, the results of Table 6 and Figure 8 support the notion that CFV functions as a single volatility factor that strongly influences the dynamics of market volatility as well as the volatility dynamics of traded factors commonly used in the asset pricing literature.

4.2. Common Factor Volatility and the Equity Premium

Numerous studies investigate the predictive power of volatility for excess stock market returns and whether volatility is a priced risk in the cross-section of equity returns. Traditional predictive regressions of excess stock returns on 'realized' measures of stock market volatility often produce inconclusive results. Table 8 contrasts the stock return forecasting power of common factor volatility (CFV) with that of traditional stock market volatility. The table reports results for the following bi-variate predictive regression at cumulative return horizons of one to 12 years:

$$R_{t+1,t+H} = \alpha + \beta dp_t + \gamma \text{VOL}_t + \epsilon_{t+H}, \tag{10}$$

[21] In unreported results, we also control for a large number of predictors that have been shown to predict market returns and market volatility. None of the factors included in Goyal and Welch (2007) have a substantial impact on the predictive power of lagged residuals on subsequent factor volatility.

where $R_{t+1,t+H}$ denotes the cumulative excess return on the CRSP value-weighted portfolio and the predictor variable dp_t is the log dividend-price ratio and the variable VOL denotes either the CFV series (results in Panel A), or stock market volatility MVOL (results in panel B). All regressions employ overlapping data over the period 1927–2017 (utilizing as many returns as possible given the horizon H). Reported t-statistics are based on Newey-West standard errors with a number of lags equal to $2 \times H - 1$. The results in Panel A demonstrate evidence of a positive risk-return trade-off when the measure of risk is common factor volatility. CFV positively and significantly forecasts excess returns at virtually all horizons considered (the t-statistic for $H = 2$ is borderline). This is notable because stock return volatility measures often are insignificant in return-forecasting regressions at short horizons. This is confirmed in Panel B, where the stock return volatility measure MVOL is borderline significant at 1 and 2 year horizons, insignificant at horizons of 3–10 years, and highly only significant at very long horizons. The forecasting power of market volatility at very long horizons is consistent with Bandi, Bretscher, and Tamoni (2018), who find evidence for even stronger long-run forecasting power for a 'backward aggregated' volatility measure that captures a low-frequency component of volatility. The R^2-values in Panel A associated with CFV exceed those at the corresponding horizon for market volatility at all horizons considered, with improvements on the order of 3–7%.

The literature concerning stock return predictability is expansive. There are a large number of competing predictors. In addition, evidence of stock return forecasting power can differ in 'out-of-sample' versus in-sample or full sample designs (Goyal and Welch (2007)), and stock return forecasting regressions can be subject to structural breaks, such that predictive power varies over time (e.g., Paye and Timmermann (2006), Farmer, Schmidt, and Timmermann (2018)). The claim we make in this context is therefore relatively narrow: we document that there appears to be stronger evidence for a positive risk-return relation in the equity market when common factor volatility is used as the measure of risk, rather than a traditional measure of realized stock market volatility.

5. Sources of Common Volatility

This section seeks to shed light on the underlying causes of common factor volatility. An obvious candidate explanation is that common variation in fundamental uncertainty drives associated commonality in stock return volatility. We consider two sources of fundamental uncertainty: macroeconomic uncertainty and uncertainty in earnings surprises. Another candidate explanation is that the sensitivity of stock returns to news is time-varying. Hence, we also consider potential sources of time-varying sensitivity including changes in operating and financial leverage and growth options.

5.1. Relations with other Uncertainty Measures

Table 9 reports statistics capturing the persistence of series and measures of co-movement between CFV and various comparison series. Panel A compares CFV with market volatility and common idiosyncratic volatility (CIV), where the latter is computed following Herskovic et al. (2016) as the cross-sectional average or monthly idiosyncratic volatility using daily residuals from a Fama-French three factor model to measure idiosyncratic returns. Panel B reports statistics for two measures of financial uncertainty, including the financial uncertainty (FINU) measure of Ludvigson et al (2018), a financial stress indicator proposed by Puttmann (2018), and the VIX and NVIX option-implied volatility measures (the latter is due to Moreira and Muir (2017)). Panel C includes several prominent measures of economic uncertainty from recent literature. These include macroeconomic uncertainty (MACU) and real economic uncertainty (REALU) measures due to Jurado et al. (2015) and Ludvigson et al (2018) and the economic policy uncertainty measure of Baker et al. (2016). Panel D provides results for a set of other financial variables often linked with notions of risk or fear. These include the tail risk measure of Kelly and Jiang (2014), denoted KJ, the default spread (DEF) and the term spread (TERM).

For each monthly series, Table 9 shows four statistics capturing the degree of persistence. These include the sample auto-correlations at lags one and six ($\hat{\phi}_1$ and $\hat{\phi}_6$), the quantity

$\hat{\phi}_1^6$, defined as the lag six autocorrelation for an AR(1) process with ϕ equal to the sample estimate, and $\hat{d}$, which is an estimate of the degree of fractional integration for the series. The right-hand portion of Table 9 presents statistics capturing the degree of co-movement of CFV with each series. These include the sample (contemporaneous) correlation between CFV and the series in levels, the sample correlation in first differences, and finally an estimate of the 'long-run' correlation between the series based on the estimation approach of Müller and Watson (2018) along with a corresponding 90% confidence set.

Previous literature documents that stock market volatility is persistent and exhibits long memory. Estimates in Table 9 confirm these characteristics and indicate that CFV, as well as CIV, are also persistent, long-memory time series. In fact, both CFV and CIV appear to be *more* persistent than (realized) market volatility. Most of the other comparison financial time series are also highly persistent series, with VRP being the least persistent among those we consider. The CFV series is highly correlated with market volatility and CIV in both levels and first differences, and the long-run correlation estimate is positive and economically, as well as statistically, significant.

CFV positive correlates with each of the financial uncertainty measures in Panel B. Among these, CFV is most highly correlated with the VIX series in levels. Changes in CFV are relatively highly correlated with changes in both the FINU measure of Jurado et al. (2015) and the VIX series. Point estimates of long-run correlations with the alternative financial uncertainty measure are positive. Although the long-run correlations are rather imprecisely estimated, and therefore often insignificant, the long-run correlation between CFV and NVIX is statistically significant. The relation between CFV and the macroeconomic uncertainty measures MACU, REALU, and EPU (Panel C) are weaker relative to the correlations with financial uncertainty measures. MACU also contains stock and bond market indices, so it isn't a pure measure of real economic uncertainty. REALU, that excludes all financial variables from MACU has a much lower correlation with CFV. Among the economic uncertainty measures, CFV shares the most affinity with the EPU measure of Baker et al. (2016). Among the additional financial variables reported in Panel D, CFV relates most closely to the default spread. However, the correlation between changes in CFV and the default spread is much lower than similar correlations for volatility and financial uncertainty measures reported in Panels A and B.

5.2. Macroeconomic tracking portfolios and underlying volatility

As an additional analysis, we compare the evidence for common volatility in the CRR macroeconomic factors with evidence for common volatility in the underlying macroeconomic series that give rise to these factors. (This is not possible using daily data because the macroeconomic series are unavailable at the daily frequency.) Both the traded return-based CRR factor set and the underlying CRR macroeconomic volatility series exhibit evidence of commonality in volatility. However, Figure 9 shows that the common component in the factor volatility has low correlations with the common component in the volatility of underlying macroeconomic series. In fact, the figure shows that the common component in CRR factor volatility moves much more closely with the common component in industry return volatility than with common component in the volatility of the underlying macroeconomic series. In untabulated results we find that the R^2s of a regression of common CRR factor volatility on the underlying common volatility is only 12.2%, while it is 50.7% when regressed on common industry volatility.

5.3. Earnings surprises

A potential concern with the tests using macroeconomic aggregates is timing. Financial markets may react today to news about macroeconomic aggregates far into the future. In other words, it is difficult to determine what macroeconomic information is a surprise at any point in time and what has already been factored into prices. To address this concern, we turn to earnings surprises. It is relatively easier to measure surprises in earnings and to isolate return reactions that are related to the surprise in earnings by focusing on a short period around a firm's earnings announcement. Our measure of the volatility of fundamental news is the volatility of earnings surprises (forecast errors relative to analyst expectations or a seasonal random walk earnings model). The volatility of the market reaction to news is the volatility of returns in the three day window centered on the earnings date.

First, we test whether there is commonality in the volatility of earnings surprises and earnings announcement returns across groups of stocks. In particular, we measure the standard deviation of earnings surprises every quarter within each Fama-French 12 industry. We estimate two sets of earnings surprises: SUE_{IBES}, in which surprises are relative to the median analyst forecast of earnings per share, and SUE_{SRW}, in which surprises are relative to a seasonal random walk model for earnings. Both measures are standardized by price and computed as in Livnat and Mendenhall (2006). SUE_{IBES} is available from 1985, while SUE_{SRW} is available from 1975. We also perform a similar computation for the standard deviation of cumulative abnormal returns (stock return minus market return) in the three day window around the earnings announcement date.

We find that there is a large degree of commonality in the (log) volatility of earnings surprises across industries. The first principal component explains about 80% of the variance of both earnings surprise volatility series. There is also commonality in the variance of earnings announcement returns, albeit somewhat smaller in magnitude. The first principal component explains about 70% of the variance of this series.

The next question is whether the commonality in variance of these series is correlated with CFV. Table 10 reports regression of CFV on the first principal component of three earnings surprise volatility series. The first 4 specifications are in levels, while the next 4 are in innovations of all left and right hand side variables. Surprisingly, the level of volatility of analyst surprises is uncorrelated with contemporaneous CFV. The volatility of surprises from a simple random walk model is correlated with CFV with an R^2 of just under 30%. In contrast the volatility of announcement returns is highly correlated with CFV with an R^2 of 72%. To assuage concerns about regressions with persistent variables, we also report regressions of innovations of these series. Innovations of all series are measured using an ARMA(1,1) filter. In the innovations regressions, the correlations of both the earnings surprise volatility series with CFV are small—R^2 are 2%. The volatility of announcement return series continues to be highly correlated with CFV with an R^2 of 61%. The correlation of earnings announcement return volatility with CFV may be a manifestation of the correlation of CIV with CFV because both are volatilities of idiosyncratic returns and hence, is perhaps not that surprising. What is noteworthy from these results is that although there appears to be a common component in the volatility of fundamental cash flow news, this component does not seem to be correlated with CFV (especially in the innovations regressions).

5.4. Explanations for time-varying sensitivity to news

The preceding results suggest that the commonality in volatility we document is largely related to financial markets rather than fundamental uncertainty. One possible source of common volatility arising in financial markets is that there is time-variation in the sensitivity of stock prices to fundamental news. The literature on financial and real options finds that options magnify the volatility of the underlying asset. For example, Galai and Masulis (1976) show that the optionality created by financial leverage amplifies the volatility of a firm's assets. Growth options and operating leverage also work in a similar manner. Thus, any common variation in real or financial options across firms could give rise to the commonality in systematic and idiosyncratic volatility we observe.

We therefore test whether CFV is correlated with the following measures of financial and operating leverage as well as growth options. Book financial leverage is short term debt plus long-term debt divided by total assets. Book operating leverage is operating profits divided by total assets as in Novy-Marx (2010). We also consider market versions of these measures, in which the denominator is market equity plus book total liabilities. Our final measure is the market-to-book ratio as a proxy for growth options. We take the average of each of these variables every quarter. Accounting variables are lined up market variables as on the fiscal quarter end-date. Panel B of Table 10 presents regressions of innovations in CFV on innovations in each of these variables. As before innovations are from an ARMA (1,1) model. We find that innovations in book operating and financial leverage have small correlations with innovations in CFV, with R^2s of 2%. The next two specifications examine market leverage and contemporaneous average stock returns. Returns are included to because innovations in market leverage are positively correlated with returns, and returns are negatively correlated with volatility. (Not including returns results in a negative sign for the leverage measures). These specifications show that the incremental explanatory power of market leverage measures is small. Finally we also see that the market-to-book ratio is not significantly related to CFV. Overall, these measures of real and financial options have small correlations with CFV suggesting that explanations that rely on optionality are unlikely to explain our findings.

6. Conclusion

We find that there is substantial common variation in the volatilities of a large set of factor, industry, and other long-short portfolios. This common factor volatility is also correlated with market and average idiosyncratic volatility. We show that this finding is unlikely to be spurious: explanations that rely on common exposures to the market, bid-ask bounce, or undiversified idiosyncratic risk are not supported by our tests. CFV is more persistent than market volatility and helps predicts the volatility of other factors even after controlling for additional lags of that factor's volatility. This appears to be due to a long-run equilibrium relationship that is similar to cointegration. Deviations of factor volatility from the level predicted by CFV tend to reverse in the future.

Although CFV is correlated with measures of economic uncertainty, its correlations with measures of uncertainty from financial markets are much higher. Similarly the correlation of CFV with the volatility of earnings surprises is low, but its correlation with the volatility of announcement returns is high. These results suggest that time-variation in the sensitivity to new information in financial markets, rather than the amount of new information plays a role in the time-variation in CFV. However, rational explanations for changing sensitivity to news such as increases in financial or operating leverage are not supported by our tests.

These results have the flavor of the findings of "excess volatility" in Shiller (1981) and

LeRoy and Porter (1981). These papers find that the volatility in market returns is too high to be justified by the volatility in future dividends. Our result is that there is substantial commonality in volatility across factors and individual stocks that does not seem to be due to commonality in the fundamental information or other rational explanations for time-variation in sensitivity to information that we consider. An alternate explanation for Shiller's results is time-variation in discount rates (see for example Schwert, 1991). Our results pose a challenge for this explanation, because it seems unlikely that time-variation in discount rates results in commonality between factor and idiosyncratic volatilities.

References

Andersen, T. G., Bollerslev, T., Diebold, F. X., Labys, P., 2003. Modeling and forecasting realized volatility. Econometrica 71, 579–625.

Aruoba, S. B., Diebold, F. X., Scotti, C., 2009. Real-time measurement of business conditions. Journal of Business & Economic Statistics 27, 417–427.

Asness, C. S., Frazzini, A., Pedersen, L. H., 2014. Quality minus junk. Review of Accounting Studies pp. 1–79.

Bai, J., 2003. Inferential theory for factor models of large dimensions. Econometrica 71, 135–171.

Baker, S. R., Bloom, N., Davis, S. J., 2016. Measuring economic policy uncertainty. The Quarterly Journal of Economics 131, 1593–1636.

Bandi, F., Bretscher, L., Tamoni, A., 2018. Long-run economic uncertainty, unpublished Manuscript.

Bansal, R., Yaron, A., 2004. Risks for the long run: A potential resolution of asset pricing puzzles. The journal of Finance 59, 1481–1509.

Barroso, P., Maio, P., 2019. The risk-return trade-off among equity factors, unpublished Manuscript.

Bauwens, L., Laurent, S., Rombouts, J. V., 2006. Multivariate garch models: a survey. Journal of applied econometrics 21, 79–109.

Bollerslev, T., Engle, R. F., Wooldridge, J. M., 1988. A capital asset pricing model with time-varying covariances. Journal of political Economy 96, 116–131.

Campbell, J. Y., Lettau, M., Malkiel, B. G., Xu, Y., 2001. Have individual stocks become more volatile? an empirical exploration of idiosyncratic risk. The Journal of Finance 56, 1–43.

Carhart, M. M., 1997. On persistence in mutual fund performance. The Journal of finance 52, 57–82.

Chamberlain, G., Rothschild, M., 1983. Arbitrage and mean-variance analysis on large asset markets. Econometrica 51, 1281–1304.

Chen, N., Roll, R., Ross, S., 1986. Ross, 1986, economic forces and the stock market. Journal of Business 59, 383–403.

Christoffersen, P., Langlois, H., 2013. The joint dynamics of equity market factors. Journal of Financial and Quantitative Analysis 48, 1371–1404.

Christoffersen, P., Lunde, A., Olesen, K. V., 2019. Factor structure in commodity futures return and volatility. Journal of Financial and Quantitative Analysis 54, 1083–1115.

Connor, G., Korajczyk, R. A., 1986. Performance measurement with the arbitrage pricing theory: A new framework for analysis. Journal of financial economics 15, 373–394.

Connor, G., Korajczyk, R. A., 1988. Risk and return in an equilibrium apt: Application of a new test methodology. Journal of financial economics 21, 255–289.

Connor, G., Korajczyk, R. A., Linton, O., 2006. The common and specific components of dynamic volatility. Journal of Econometrics 132, 231–255.

Cooper, I., Priestley, R., 2011. Real investment and risk dynamics. Journal of Financial Economics 101, 182–205.

Corsi, F., 2009. A simple approximate long-memory model of realized volatility. Journal of Financial Econometrics 7, 174–196.

Diebold, F. X., Nerlove, M., 1989. The dynamics of exchange rate volatility: a multivariate latent factor arch model. Journal of Applied econometrics 4, 1–21.

Duarte, J., Kamara, A., Siegel, S., Sun, C., 2014. The systematic risk of idiosyncratic volatility, unpublished Manuscript.

Engle, R. F., Granger, C. W., 1987. Co-integration and error correction: representation, estimation, and testing. Econometrica: journal of the Econometric Society pp. 251–276.

Engle, R. F., Kozicki, S., 1993. Testing for common features. Journal of Business & Economic Statistics 11, 369–380.

Engle, R. F., Marcucci, J., 2006. A long-run pure variance common features model for the common volatilities of the dow jones. Journal of Econometrics 132, 7–42.

Engle, R. F., Ng, V. K., Rothschild, M., 1990. Asset pricing with a factor-arch covariance structure: Empirical estimates for treasury bills. Journal of Econometrics 45, 213–237.

Engle, R. F., Rangel, J. G., 2008. The spline-garch model for low-frequency volatility and its global macroeconomic causes. The Review of Financial Studies 21, 1187–1222.

Fama, E. F., French, K. R., 1993. Common risk factors in the returns on stocks and bonds. Journal of financial economics 33, 3–56.

Fama, E. F., French, K. R., 2015. A five-factor asset pricing model. Journal of Financial Economics 116, 1–22.

Frazzini, A., Pedersen, L. H., 2014. Betting against beta. Journal of Financial Economics 111, 1–25.

Galai, D., Masulis, R. W., 1976. The option pricing model and the risk factor of stock. Journal of Financial economics 3, 53–81.

Goyal, A., Welch, I., 2007. A comprehensive look at the empirical performance of equity premium prediction. The Review of Financial Studies 21, 1455–1508.

Green, J., Hand, J. R., Zhang, X. F., 2017. The characteristics that provide independent information about average us monthly stock returns. The Review of Financial Studies 30, 4389–4436.

Herskovic, B., Kelly, B., Lustig, H., Van Nieuwerburgh, S., 2016. The common factor in idiosyncratic volatility: Quantitative asset pricing implications. Journal of Financial Economics 119, 249–283.

Hou, K., Xue, C., Zhang, L., 2015. Digesting anomalies: An investment approach. The Review of Financial Studies 28, 650–705.

Jurado, K., Ludvigson, S. C., Ng, S., 2015. Measuring uncertainty. American Economic Review 105, 1177–1216.

Kelly, B., Jiang, H., 2014. Tail risk and asset prices. The Review of Financial Studies 27, 2841–2871.

Kozak, S., Nagel, S., Santosh, S., forthcoming. Shrinking the cross section. Journal of Financial Economics .

LeRoy, S. F., Porter, R. D., 1981. The present-value relation: Tests based on implied variance bounds. Econometrica pp. 555–574.

Lettau, M., Pelger, M., forthcoming. Estimating latent asset-pricing factors. Journal of Econometrics .

Livnat, J., Mendenhall, R. R., 2006. Comparing the post–earnings announcement drift for surprises calculated from analyst and time series forecasts. Journal of accounting research 44, 177–205.

Moreira, A., Muir, T., 2017. Volatility-managed portfolios. The Journal of Finance 72, 1611–1644.

Müller, U. K., Watson, M. W., 2018. Long-run covariability. Econometrica 86, 775–804.

Novy-Marx, R., 2010. Operating leverage. Review of Finance 15, 103–134.

Pástor, L., Stambaugh, R. F., 2003. Liquidity risk and expected stock returns. Journal of Political Economy 111, 642–685.

Paye, B. S., Timmermann, A., 2006. Instability of return prediction models. Journal of Empirical Finance 13, 274–315.

Puttmann, L., 2018. Patterns of panic: Financial crisis language in historical newspapers. Tech. rep., University of Bonn.

Ross, S., 1976. The arbitrage theory of capital asset pricing. Journal of Finance 13, 341–360.

Schwert, G. W., 1989. Why does stock market volatility change over time? The journal of Finance 44, 1115–1153.

Schwert, G. W., 1991. Review of market volatility by robert j. shiller. Journal of Portfolio Management 17, 74–78.

Shiller, R. J., 1981. Do stock prices move too much to be justified by subsequent changes in dividends? The American Economic Review 71, 421–436.

Stambaugh, R. F., Yuan, Y., 2016. Mispricing factors. The Review of Financial Studies 30, 1270–1315.

Appendix A. Beta Estimation Error

To illustrate the concern, consider a set of portfolio (or individual security) returns with potentially time-varying market exposure. Let t index lower frequency periods (e.g. months or quarters) and d index trading days, with the notation $d \in t$ representing shorthand for the set of trading days within the t-th period. Suppose daily returns for N portfolios (or securities) evolve according to a version of the CAPM with time-varying betas, as follows:

$$r_{i,d} = \beta_{i,d}\, r_{m,d} + \epsilon_{i,d}, \tag{11}$$

where $r_{i,d}$ and $r_{m,d}$ denote daily returns in excess of the risk-free rate for the i-th portfolio and the market, respectively. We assume the daily return shocks $\epsilon_{i,d}$ are uncorrelated cross-sectionally and over time, with time-invariant volatility σ_{ϵ_i}. Portfolio market betas $\beta_{i,d}$ are potentially time-varying. By construction, there is no time-varying common component in residual volatility across portfolios in this stylized setting.

Now, suppose that an econometrician seeks to measure the common factor variance, which we define as $CFV_t^2 = (1/N)\sum_{i=1}^{N}\sum_{d\in t}\epsilon_{i,d}^2$, based on the sum of squared residual estimates from the market model.[22] Specifically, the econometrician constructs a proxy as

$$\widehat{CFV}_t^2 = (1/N)\sum_{i=1}^{N}\sum_{d\in t}\hat{\epsilon}_{i,d}^2 = (1/N)\sum_{i=1}^{N}\sum_{d\in t}\left(\epsilon_{i,d} + (\beta_{i,d} - \hat{\beta}_{i,d})r_{m,d}\right)^2, \tag{12}$$

where $\hat{\beta_{i,t}}$ denotes an estimate of the true security beta for the corresponding trading day. It then follows that

$$\widehat{CFV}_t^2 \approx CFV_t^2 + \sum_{d\in t} r_{m,d}^2\,\overline{(\hat{\beta}_{i,d} - \beta_{i,d})^2}, \tag{13}$$

where the overline is short-hand for the cross-sectional average, i.e., $\overline{X_i} \equiv (1/N)\sum_{i=1}^{N} X_i$. Equation (13) shows that the common residual variance computed based on the residuals from a market model regression mechanically contains a component related to market variance when daily betas are estimated with error.

$$\widehat{CFV}_t^2 \approx CFV_t^2 + RV_{m,t}^2\,\overline{(\hat{\beta}_{i,t} - \beta_{i,t})^2} + WRV_{m,t}^2, \tag{14}$$

where CFV_t^2 denotes the true average of the within-period sum of squared daily residuals across assets, $RV_{m,t}^2$ denotes the 'realized variance' for the market (sum of within-period squared market excess returns), $\hat{\beta}_{i,t}$ is the estimated beta used to adjust within-period daily

[22] It is convenient to discuss the implications of beta measurement error in terms of variances rather than volatilities. We adopt the notation CFV^2 for common factor variance to distinguish it from common factor *volatility* CFV that is emphasized in our empirical work.

returns, $\beta_{i,t}$ denotes the average daily beta value within period t, and the overline symbol indicates the average across assets. The final term $WRV_{m,t}^2$ denotes a weighted version of the realized market variance with daily weights equal to $\overline{(\beta_{i,d} - \beta_{i,t})^2}$.

Table 1: **Volatility Commonality for Factor and Anomaly Portfolios**

The table shows statistics associated with quarterly volatility measures constructed from market-adjusted factor returns. Daily portfolio returns are regressed on daily realization of the market excess return factor to produce market-adjusted returns. Separate regressions are performed each calendar year. Quarterly volatility series for each factor are computed by summing squared daily residuals within each quarter. The common component of volatility is defined as the first principal component of the set of market-adjusted volatility series. The first column specifies the set of factors. The second column indicates the sample period. N equals the number of factors in the corresponding set. % Expl. shows the percent of total variance explained by CFV. $\hat{\rho}_{\text{MKT}}$ denotes the time-series sample correlation between FV and market volatility. $\hat{\rho}_{\text{ADS}}$ denotes the sample correlation of FV with the business conditions measure of (Aruoba, Diebold, and Scotti (2009)). $\overline{R^2}$ denotes the pooled mean of the R^2-values for daily market model regressions. $\overline{\hat{\beta}}$ denotes the pooled mean of market beta estimates used to obtain residual returns across portfolios and years. Similarly, $\overline{\text{SE}(\hat{\beta})}$ denotes the pooled mean of the heteroskedasticity-robust standard error of the market beta estimates.

Portfolios	Sample	N	% Expl.	$\hat{\rho}_{\text{MKT}}$	$\hat{\rho}_{\text{ADS}}$	$\overline{\hat{\beta}}$	$\overline{\text{SE}(\hat{\beta})}$	$\overline{R^2}$
Panel A: Characteristics-Based Factors								
SMB, HML, UMD, BAB	1945	4	81.99	0.84	-0.46	-0.08	0.04	25.16
FF 5 + UMD	1963	5	76.71	0.79	-0.43	-0.06	0.03	20.34
FF5 + UMD, MGMT, PRF	1963	7	77.30	0.81	-0.47	-0.08	0.04	21.34
Previous Row + BAB, QMJ	1963	9	78.61	0.83	-0.48	-0.11	0.03	22.09
Panel B: Anomaly Long-Short Portfolios								
GHZ Anomalies	1964	62	71.31	0.75	-0.38	0.10	0.05	17.59
KNS Anomalies	1963	41	81.32	0.68	-0.36	0.06	0.07	20.59
Panel C: Industry Portfolios								
12 VW Industries	1926	12	70.92	0.79	-0.37	0.96	0.04	71.30
30 VW Industries	1926	30	63.66	0.84	-0.39	0.96	0.06	58.60
Panel D: Macroeconomic Factors								
CRR Factors	1963	7	84.81	0.69	-0.33	0.17	0.10	5.33
CRR Small	1963	5	87.46	0.63	-0.34	0.10	0.07	5.66
Panel E: Statistical Factors								
Statistical Factors: FF+IND	1963	10	76.43	0.80	-0.45	0.09	0.09	20.45
Statistical Factors: GHZ+IND	1964	10	76.84	0.78	-0.43	0.05	0.09	20.31
Statistical Factors: KNS+IND	1963	10	76.33	0.76	-0.42	0.14	0.08	22.17

Table 2: **Commonality in Correlations**

The table shows statistics associated with quarterly correlation measures for factor and anomaly portfolios. The first column specifies the set of factors. The second column indicates the sample period. N equals the number of factors in the corresponding set. The column (% Expl.) shows the percent of total variance explained by the first principal component extracted from the quarterly correlation series for the set of portfolios. $\hat{\rho}_{MKT}$ denotes the time-series sample correlation between the average correlation and market volatility, where the latter is computed as squared daily within-period market factor returns. $\hat{\rho}_{ADS}$ denotes the time-series sample correlation between the average correlation and the ADS measure of economic conditions. Results are shown both for raw correlations (left portion of the table) and for correlations computed using daily residuals from a market model regressions estimated each calendar year (right portion of the table).

			Correl. Factor (Raw)			Correl. Factor (Residuals)		
Portfolios	Sample	N	% Expl.	$\hat{\rho}_{MKT}$	$\hat{\rho}_{ADS}$	% Expl.	$\hat{\rho}_{MKT}$	$\hat{\rho}_{ADS}$
Panel A: Characteristics-Based Factors								
SMB, HML, UMD, BAB	1945	4	25.56	0.15	-0.15	26.24	0.15	-0.32
FF 5 + UMD	1963	5	30.62	-0.07	-0.15	26.96	0.17	0.02
FF5 + UMD, MGMT, PRF	1963	7	28.32	0.14	-0.27	23.63	0.11	-0.09
Previous Row + BAB, QMJ	1963	9	23.85	0.31	-0.33	18.54	0.30	-0.22
Panel B: Anomaly Long-Short Portfolios								
GHZ Anomalies	1964	62	10.19	-0.17	0.30	11.40	0.28	-0.14
KNS Anomalies	1963	41	17.82	-0.17	0.22	15.23	-0.15	0.05
Panel C: Industry Portfolios								
12 VW Industries	1926	12	59.23	0.49	-0.17	15.72	0.13	-0.18
30 VW Industries	1926	30	49.15	0.43	-0.21	12.24	0.10	-0.24
Panel D: Macroeconomic Factors								
CRR Factors	1963	7	14.16	-0.10	-0.04	16.20	-0.16	0.06
CRR Small	1963	5	21.02	-0.11	0.01	21.98	-0.22	0.03
Panel E: Statistical Factors (Excluding 'Market')								
Statistical Factors: FF+IND	1963	10	12.21	-0.04	0.02	13.10	0.05	0.05
Statistical Factors: GHZ+IND	1964	10	12.86	0.14	-0.45	13.98	-0.28	0.26
Statistical Factors: KNS+IND	1963	10	13.44	0.00	0.12	12.43	0.24	-0.16

Table 3: **Volatility Commonality for Factor and Anomaly Portfolios: Robustness**

This table presents robustness checks for results in Table 1 concerning the common factor volatility (CFV) extracted from factor and anomaly portfolios. The method for computing CFV is the same as that described in the header for Table 1 except as explicitly indicated in the various panels of this table. Specifically, Panel A presents results for selected sets of portfolios when market model regressions are estimated each month, rather than each year as in Table 1. To conserve space, we report only certain representative factor sets. 'Characteristics-Based Factors' refers to full set of such factors ('Previous row + BAB and QMJ' in Table 1). 'CRR Factors' set refers to the CRR tracking portfolios excluding oil and consumption growth. The '10 Statistical Factors' set of portfolios corresponds to the 'Long set' from Table 1. The 'Anomalies' set of portfolios refers to the 'wide set' of 92 anomaly portfolios covering 1980–2018. Panel B shows results with no market adjustment ($\hat{\beta} = 0$) for statistical factors and a set of randomly generated anomaly portfolios as described in the main text. Because beta estimates are fixed at zero we do not report standard errors or R^2-values for this panel. Panel C shows results for equal-weighted, as opposed to value-weighted, Fama-French industry portfolio returns. Panel D shows results for Fama-French factor returns computed using daily closing bid and ask prices from CRSP. See Table 1 description for definitions of reported statistics.

Portfolios	Sample	N	% Expl.	$\hat{\rho}_{\text{MKT}}$	$\hat{\rho}_{\text{ADS}}$	$\overline{\hat{\beta}}$	$\overline{\text{SE}(\hat{\beta})}$	$\overline{R^2}$
Panel A: Monthly market model regressions								
Characteristics-Based Factors	1963	9	78.54	0.83	-0.48	-0.12	0.09	27.86
GHZ Anomalies	1964	62	71.31	0.75	-0.37	0.10	0.14	23.51
12 VW Industries	1926	12	70.58	0.79	-0.36	0.95	0.12	69.82
CRR Small	1963	5	87.48	0.62	-0.32	0.10	0.21	11.38
Statistical Factors: GHZ+IND	1964	10	76.43	0.78	-0.43	0.05	0.23	26.85
Panel B: No Market Adjustment ($\hat{\beta} = 0$)								
Characteristics-Based Factors	1963	9	74.25	0.84	-0.50	0.00	0.00	-
GHZ Anomalies	1964	62	72.90	0.77	-0.39	0.00	0.00	-
Statistical Factors: SCS+Ind	1973	10	70.43	0.77	-0.38	0.00	0.00	-
Panel C: Equal-Weighted Industry Portfolios								
12 EW Industries	1926	12	78.52	0.87	-0.40	0.89	0.05	64.63
30 EW Industries	1926	30	72.60	0.84	-0.44	0.87	0.07	52.23
Panel D: Returns Constructed from Closing Bid and Ask								
SMB, HML, CMA, RMW, UMD	1993	5	73.84	0.48	-0.36	-0.06	0.04	16.71

Table 4: **Common Factor Volatility: Monthly Data**

The table shows statistics associated with volatility measures constructed from monthly market-adjusted factor returns. Market-adjusted returns are based on rolling 60-month regressions of monthly factor returns on the monthly market excess return. Monthly volatility series for each factor are computed as rolling sums of squared daily residuals over the past 12 months. The common factor volatility (CFV) measure equals the first principal component of the set of market-adjusted volatility series. The first column specifies the set of factors. The second column indicates the sample period. N equals the number of factors in the corresponding set. The column (% Expl.) shows the percent of total variance explained by CFV. $\hat{\rho}_{\text{MKT}}$ denotes the time-series sample correlation between CFV and market volatility, where the latter is computed as squared daily within-period market factor returns. $\hat{\rho}_{\text{ADS}}$ denotes the sample correlation of FV with the business conditions measure of (Aruoba et al. (2009)). $\overline{R^2}$ denotes the pooled mean of the R^2-values for the market model regressions. $\overline{\hat{\beta}}$ denotes the pooled mean of market beta estimates used to obtain residual returns across portfolios and years. Similarly, $\overline{\text{SE}(\hat{\beta})}$ denotes the pooled mean of the heteroskedasticity-robust standard error of the market beta estimates.

Portfolios	Sample	N	% Expl.	$\hat{\rho}_{\text{MKT}}$	$\hat{\rho}_{\text{ADS}}$	$\overline{\hat{\beta}}$	$\overline{\text{SE}(\hat{\beta})}$	$\overline{R^2}$
Panel A: Characteristics-Based Factors								
SMB, HML, UMD, BAB	1930	4	77.99	0.74	-0.25	0.00	0.10	14.15
FF 5 + UMD	1963	5	70.52	0.50	-0.24	-0.06	0.09	13.77
FF5 + UMD, MGMT, PRF	1963	7	67.83	0.52	-0.29	-0.11	0.09	16.16
Previous Row + BAB, QMJ	1963	9	65.94	0.55	-0.28	-0.12	0.09	16.94
Panel B: Anomaly Long-Short Portfolios								
GHZ Anomalies (wide set)	1980	93	58.92	0.50	-0.18	0.08	0.11	14.85
KNS Anomalies	1963	43	69.64	0.46	-0.22	0.07	0.12	20.40
Panel C: Industry Portfolios								
12 VW Industries	1927	12	54.09	0.67	-0.26	0.96	0.08	71.96
30 VW Industries	1927	30	46.73	0.70	-0.26	1.01	0.10	64.10
Panel D: Macroeconomic Factors								
CRR Small	1963	5	51.86	0.45	-0.30	0.04	0.10	6.24
CRR Underlying	1963	5	41.55	0.34	-0.36	-0.00	0.02	3.63
Panel E: Statistical Factors								
Statistical Factors: FF+IND	1963	10	56.32	0.57	-0.28	0.05	0.17	17.98
Statistical Factors: GHZ + IND	1980	10	68.79	0.51	-0.25	0.12	0.16	20.06
Statistical Factors: KNS + IND	1963	10	66.32	0.54	-0.29	0.02	0.16	23.04

Table 5: **Fitted Factor Volatilities**

This table shows the contemporaneous relations between monthly volatilities of the the five Fama and French (2015) factors and UMD from Carhart (1997) with log(CFV) for the sample period July 1967 through December 2018. Factor volatilities are measured using all daily returns for each factor within a month. Panel A reports the results of contemporaneous ols regressions of natural logarithms of the volatilities of each traded factor on the natural log of CFV:

$$log(\sigma_{F_k,t}) = a + b \cdot log(CFV_t) + e_t.$$

OLS t-statistics are reported in parentheses. Panel B reports the correlations of log factor volatilities, $log(\sigma_{F_k,t})$. Panel C reports the correlation matrix of the residuals from the regressions reported in Panel A.

Panel A: Regression Results

	Mrktrf	SMB	HML	RMW	CMA	UMD
log(CFV)	0.59	0.54	0.70	0.78	0.73	0.69
	(18.71)	(16.36)	(25.51)	(32.21)	(27.86)	(24.24)
R^2	0.35	0.29	0.49	0.61	0.54	0.47
N	665	665	665	665	665	665

Panel B: Raw Volatility Correlations

Mrktrf	1	0.72	0.62	0.55	0.52	0.62
SMB	0.72	1	0.54	0.50	0.50	0.55
HML	0.62	0.54	1	0.64	0.73	0.66
RMW	0.55	0.50	0.64	1	0.68	0.69
CMA	0.52	0.50	0.73	0.68	1	0.59
UMD	0.62	0.55	0.66	0.69	0.59	1

Panel C: Residual Volatility Correlations

Mrktrf	1	0.60	0.35	0.18	0.16	0.37
SMB	0.60	1	0.27	0.16	0.19	0.30
HML	0.35	0.27	1	0.21	0.44	0.35
RMW	0.18	0.16	0.21	1	0.26	0.34
CMA	0.16	0.19	0.44	0.26	1	0.17
UMD	0.37	0.30	0.35	0.34	0.17	1

Table 6: **Predicting Factor Volatilities**

The table shows the results of predictive regressions of monthly (log) volatilities for the five Fama and French (2015) factors and UMD from Carhart (1997) for the sample period July 1967 through December 2018. For each factor, we predict the logarithm of volatility one month in the future. Our regression specification is given by:

$$log(FVOL_{t+1}) = a + b'\mathbf{X}_t + \gamma'\mathbf{Z} + e_{t+1},$$

where $FVOL_t$ is the given factor's volatility measured over month t. $\mathbf{X}$ is a d-dimensional vector of predictors where $d \in \{1, 2\}$ with elements of $\mathbf{X}$ coming from the set of random variables $\{log(FVOL_t), log(CFV_t), resid_t\}$, where $resid_t$ denotes the given factor's residual from the regression described in Table 5. The vector $\mathbf{Z}$ includes additional lags of factor volatilities over the past 3 months and over the past 12 months. Specifications (4) and (8) include the additional lags in the fashion of an HAR model of Corsi (2009): $\mathbf{Z} = \{log(FVOL_{t-2,t}), log(FVOL_{t-11,t})\}$ in addition to $log(FVOL_t)$. Newey-West t-statistics with 12 month lags are reported in parentheses.

	Panel A: MRKTRF VOL				**Panel B: SMB VOL**			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
intercept	0.01	0.01	0.00	-0.08	0.00	0.00	0.00	-0.09
	(0.12)	(0.07)	(0.13)	(-2.01)	(0.04)	(0.07)	(0.06)	(-2.09)
resid			-0.31	-0.26			-0.31	-0.26
			(-6.61)	(-7.07)			(-4.03)	(-3.59)
$log(FVOL)$	0.66		0.87	0.59	0.56		0.79	0.60
	(16.78)		(19.75)	(9.25)	(14.01)		(12.90)	(7.14)
$log(CFV)$		0.51				0.42		
		(8.80)				(8.00)		
R^2	0.44	0.26	0.46	0.48	0.32	0.18	0.34	0.36
N	660	660	660	654	660	660	660	654
HAR Lags	no	no	no	yes	no	no	no	yes

	Panel C: HML VOL				**Panel D: RMW VOL**			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
intercept	0.00	0.00	0.00	-0.09	- 0.00	0.00	-0.00	-0.10
	(0.07)	(0.07)	(0.09)	(-2.77)	(-0.03)	(0.01)	(-0.01)	(-4.32)
resid			-0.27	-0.21			-0.35	-0.25
			(-4.56)	(-3.79)			(-4.75)	(-3.97)
$log(FVOL)$	0.71		0.84	0.49	0.74		0.88	0.39
	(16.55)		(17.09)	(7.70)	(16.10)		(17.34)	(5.89)
$log(CFV)$		0.60				0.68		
		(9.89)				(11.48)		
R^2	0.50	0.35	0.52	0.55	0.55	0.47	0.56	0.63
N	660	660	660	654	660	660	660	654
HAR Lags	no	no	no	yes	no	no	no	yes

Table 7: **Predicting Factor Volatilities (continued)**

	Panel E: CMA VOL				Panel F: UMD VOL			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
intercept	0.00	0.00	0.00	-0.08	-0.00	0.00	-0.00	-0.05
	(0.01)	(0.03)	(0.03)	(-2.88)	(-0.03)	(0.00)	(-0.02)	(-1.53)
$resid$			-0.30	-0.20			-0.28	-0.23
			(-4.62)	(-3.42)			(-5.74)	(-5.06)
$log(FVOL)$	0.72		0.86	0.42	0.73		0.86	0.65
	(15.40)		(15.39)	(7.26)	(21.37)		(20.85)	(10.21)
$log(CFV)$		0.64				0.60		
		(9.18)				(9.96)		
R^2	0.53	0.40	0.55	0.59	0.54	0.36	0.56	0.57
N	660	660	660	654	660	660	660	654
HAR Lags	no	no	no	yes	no	no	no	yes

Table 8: **Return Forecasting Regressions**

The table analyzes the stock-return forecasting information associated with the common factor volatility measure (CFV). The dependent variable is the cumulative excess return on the market portfolio over H periods, denoted $R_{t+1,t+H}$. The regression model is

$$R_{t+1,t+H} = \alpha + \beta dp_t + \gamma \text{VOL}_t + \epsilon_{t+H},$$

where the predictor variable dp_t is the log dividend-price ratio and the variable VOL denotes either the CFV series (results in Panel A), or stock market volatility MVOL (results in panel B). All regressions employ overlapping data. Reported t-statistics are based on Newey-West standard errors with a number of lags equal to $2 \times H - 1$. The sample begins in 1927 and ends with the last year over which the H-period ahead excess return can be computed given that the data end in 2017. For example, at the four-year horizon ($H = 4$), the sample period is 1927–2014, so that the final four-year excess return covers the period 2014–2017.

H (years)	1	2	3	4	6	8	10	12
Panel A: Common Factor Volatility (CFV)								
$\hat{\beta}$	0.17	0.31	0.38	0.49	0.98	1.64	2.38	3.58
t-statistic	3.46	3.68	3.63	4.04	5.18	5.90	6.08	6.75
$\hat{\gamma}$	0.08	0.13	0.11	0.15	0.33	0.48	0.75	1.36
t-stat	2.59	2.58	1.50	2.01	2.64	4.24	3.05	5.28
R^2-value (%)	13.96	21.29	20.43	22.41	37.77	49.60	55.55	59.88
Panel B: Stock Market Volatility (MVOL)								
$\hat{\beta}$	0.14	0.28	0.35	0.44	0.89	1.50	2.20	3.14
t-statistic	2.96	3.22	3.29	3.33	4.30	4.86	5.76	5.23
$\hat{\gamma}$	0.04	0.08	0.05	0.07	0.20	0.31	0.59	0.81
t-stat	1.66	1.84	0.77	0.77	1.55	3.42	3.99	6.57
R^2-value (%)	11.00	19.27	18.69	20.31	35.88	48.03	56.96	55.29

Table 9: **Common Volatility: Relations with other Uncertainty Measures**

The table shows statistics pertaining to the persistence and correlations between CFV and a set of volatility, uncertainty, and financial variables. $\hat{\phi}_1$ and $\hat{\phi}_6$ denote sample autocorrelations at lags one and six, respectively. $\hat{\phi}_1^6$ equals the implied correlation at lag six assuming that the variable follows a covariance stationary AR(1) process with AR(1) coefficient equal to the sample estimate $\hat{\phi}_1$. $\hat{d}$ denotes an estimate of the long-memory parameter d associated with each series based on the Exact Whittle method. $\hat{\rho}(CFV)$ is the sample correlation of the corresponding variable with CFV. $\hat{\rho}(\Delta FV)$ denotes the sample correlation of *changes* in the corresponding variable with changes in CFV. $\hat{\rho}_{\mathrm{LR}}(CFV)$ is an estimate of the long-run correlation between the corresponding variable and FV using the method of Müller and Watson (2018), and the table provides the associated 90% confidence sets for this parameter. Panel A presents results for our main equity volatility measures, including common factor volatility (CFV), market volatility (MVOL), and common idiosyncratic volatility (CIV). Panels B and C cover alternative financial and economic uncertainty measures, respectively. Panel D shows results for other benchmark financial variables. Long-run correlation estimates are omitted for the variables VIX and VRP due to insufficient sample length.

	Persistence Measures				Co-Movement Measures			
Variable	$\hat{\phi}_1$	$\hat{\phi}_6$	$\hat{\phi}_1^6$	$\hat{d}$	$\hat{\rho}(CFV)$	$\hat{\rho}(\Delta CFV)$	$\hat{\rho}_{\mathrm{LR}}(CFV)$	90% CS
Panel A: Main Equity Volatility Measures								
CFV	0.83	0.61	0.32	0.69	1.00	1.00	1.00	-
MVOL	0.67	0.40	0.09	0.52	0.86	0.76	0.90	[0.71 0.97]
CIV	0.91	0.62	0.58	0.72	0.88	0.79	0.87	[0.65 0.96]
Panel B: Financial Uncertainty Measures								
FISTR	0.85	0.73	0.37	0.72	0.43	0.15	0.40	[-0.03 0.71]
FINU	0.92	0.25	0.60	0.80	0.57	0.59	0.13	[-0.41 0.65]
NVIX	0.83	0.52	0.32	0.61	0.57	0.36	0.49	[0.01 0.81]
VIX	0.64	0.19	0.07	0.55	0.73	0.60	0.00	-
Panel C: Economic Uncertainty Measures								
MACU	0.95	0.44	0.74	0.91	0.37	0.31	-0.00	[-0.51 0.50]
REALU	0.91	0.42	0.56	0.56	0.08	0.18	-0.27	[-0.71 0.34]
EPU	0.69	0.54	0.10	0.64	0.43	0.28	0.42	[-0.01 0.72]
Panel D: Other Financial Variables								
KJ	0.92	0.70	0.61	1.05	-0.05	-0.20	-0.12	[-0.67 0.48]
VRP	0.25	0.11	0.00	0.35	-0.10	-0.31	0.00	-
DEF	0.90	0.61	0.52	0.64	0.74	0.17	0.71	[0.34 0.90]
TERM	0.85	0.39	0.39	0.63	0.21	-0.05	0.25	[-0.27 0.64]

Table 10: **Potential explanations for the commonality in volatility**

This table examines potential explanations for the commonality in volatility. Panel A presents regressions of log CFV on measures of volatility derived from earnings announcements. In each quarter, we estimate earnings surprises relative to median analyst expectations (SUE_{IBES}) and a seasonal random walk model for earnings (SUE_{SRW}). $\sigma^2(SUE)$ is the first principal component of log variances of earnings surprises within each Fama-French 12 industry. $\sigma^2(CAR)$ is the first principal component of the log variance of the cumulative abnormal (stock - market) return in the three day earnings announcement window within each Fama-French 12 industry. The first four specifications are in levels. The next four specifications are in innovations of all left and right hand side variables. Innovations are measured from an ARMA(1,1) model for each series. Panel B presents results of regressions of innovations in CFV on innovations in measures of financial and real options. Book (Mkt) leverage is total long and short term debt divided by book (market) value of total assets. Book (Mkt) operating leverage is operating expense divided by book (market) total assets, Market total assets are market equity + total assets - book equity. M/B is the ratio of market to book equity. All financial variables are averages across all firms reporting earnings in the quarter for which volatility is measured. Returns are the average returns of all firms in the quarter over which volatility is measured. Standard errors are Newey-West with 12 lags for regressions in levels and 3 for innovations.

Panel A: Earnings surprise and announcement return volatility

	Levels				Innovations			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Intercept	-3.18	-3.25	-3.25	-3.25	-0.00	-0.00	0.00	-0.00
	(-49.24)	(-59.76)	(-106.35)	(-107.57)	(-0.05)	(-0.05)	(-0.01)	(-0.02)
$\sigma^2(SUE_{IBES})$	0.00				0.03			
	(0.12)				(2.94)			
$\sigma^2(SUE_{SRW})$		0.05		0.01		0.02		0.00
		(4.34)		(0.77)		(2.05)		(0.81)
$\sigma^2(CAR)$			0.10	0.10			0.12	0.12
			(10.27)	(10.07)			(16.16)	(15.45)
R^2	0.00	0.18	0.71	0.71	0.04	0.03	0.62	0.62
N	136	176	176	176	136	176	176	176

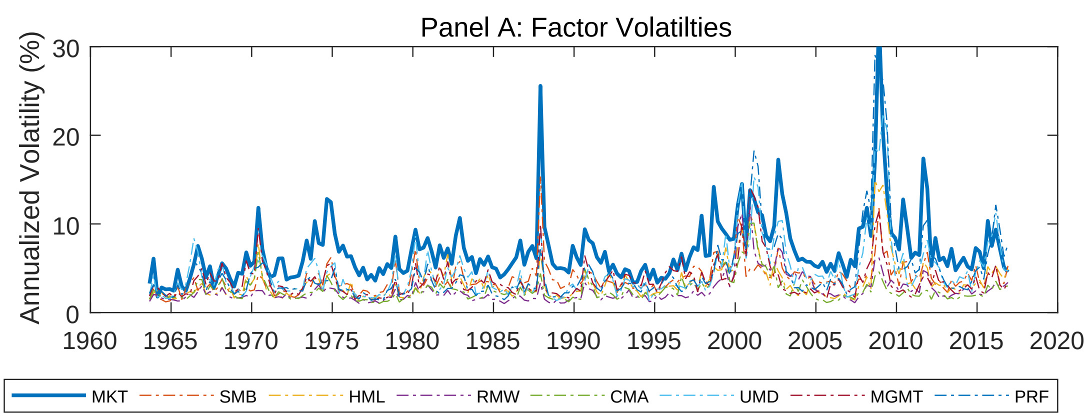
Table 10: **Potential explanations for the commonality in volatility (continued.)**

Panel B: Real and financial options

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	-0.00	-0.00	0.03	0.02	0.03	-0.05	-0.05
	(-0.04)	(-0.03)	(1.48)	(1.08)	(1.36)	(-0.92)	(-0.95)
Financial Leverage (Book)	2.13						1.43
	(0.98)						(0.82)
Operating Leverage (Book)		0.59					0.11
		(2.16)					(0.27)
Returns			-0.83	-0.58	-0.78	-0.85	-0.84
			(-3.46)	(-2.22)	(-3.04)	(-3.55)	(-3.47)
M/B						0.03	0.03
						(1.71)	(1.72)
Financial Leverage (Mkt)				3.55			
				(1.58)			
Operating Leverage (Mkt)					0.25		
					(0.86)		
R^2	0.01	0.01	0.14	0.16	0.15	0.15	0.16
N	188	188	188	188	188	188	188

Fig. 1: Commonality in Characteristics-Based Factor Return Volatilities

The figure plots quarterly time series of volatility measures for common asset pricing factors. Panel A displays quarterly volatility series for seven asset pricing factor returns. These include the market excess return (MKT), the Fama-French size (SMB), value (HML) factors, investment (RMW) and profitability (CMA) factors, along with the management (MGMT) and performance (PRF) mispricing factors of Stambaugh and Yuan (2016). Raw variances are computed as the sum of squared daily returns for each factor during the corresponding quarter. The plot depicts the associated volatility on an annualized basis. Panel B depicts the volatility of the residuals of various long-short factors with respect to market returns. Daily residual returns for each factor are based on a single factor (market) model regression estimated over each calendar year. Annualized residual volatility measures for each quarter are computed based on the sum of squared daily residuals. Panel B also depicts a measure of common volatility ('Avg. Vol.'), computed as the (equal-weighted) average of the individual market-adjusted factor volatilities.



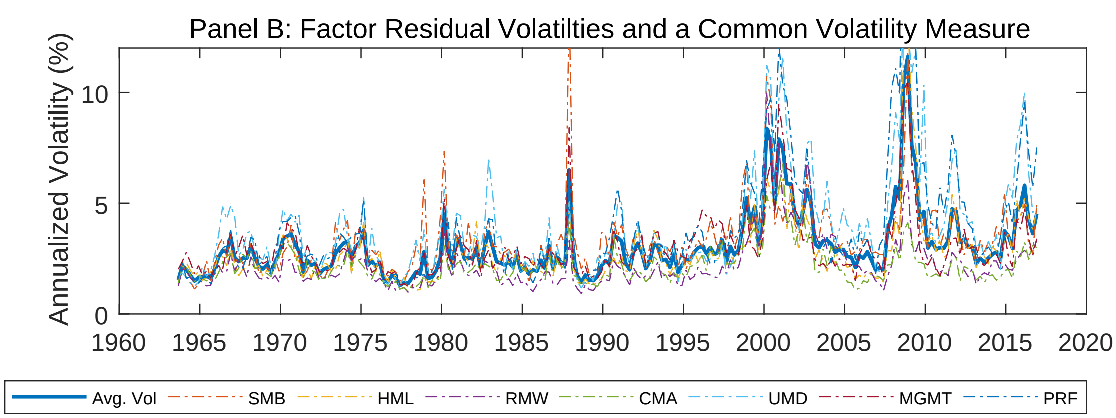


Fig. 2: Industry and Anomaly Return Volatilities

This figure plots quarterly time series of market-adjusted volatility measures for a set of industry portfolios and a set of long-short anomaly portfolios. Panel A displays time series of quarterly volatility measures for the market-adjusted returns for the Fama-French 12 value-weighted industry portfolios. Volatility measures are based on the sum of squared daily returns within the corresponding quarter and are annualized. Panel A also depicts a common volatility measure computed as the (equal-weighted) average of the individual market-adjusted factor volatilities. Panel B plots quarterly volatility measures for raw returns for a randomly selected subset of 25 among the anomaly portfolios analyzed in the paper. Anomaly market-adjusted return volatilities are displayed as colored dashed-dot lines. The figure also shows a common volatility measure for these series computed as the (equal-weighted) average of the individual anomaly volatility series. Daily market-adjusted returns for both industries and anomaly long-short portfolios are based on a single factor (market) model regression estimated over each calendar year.

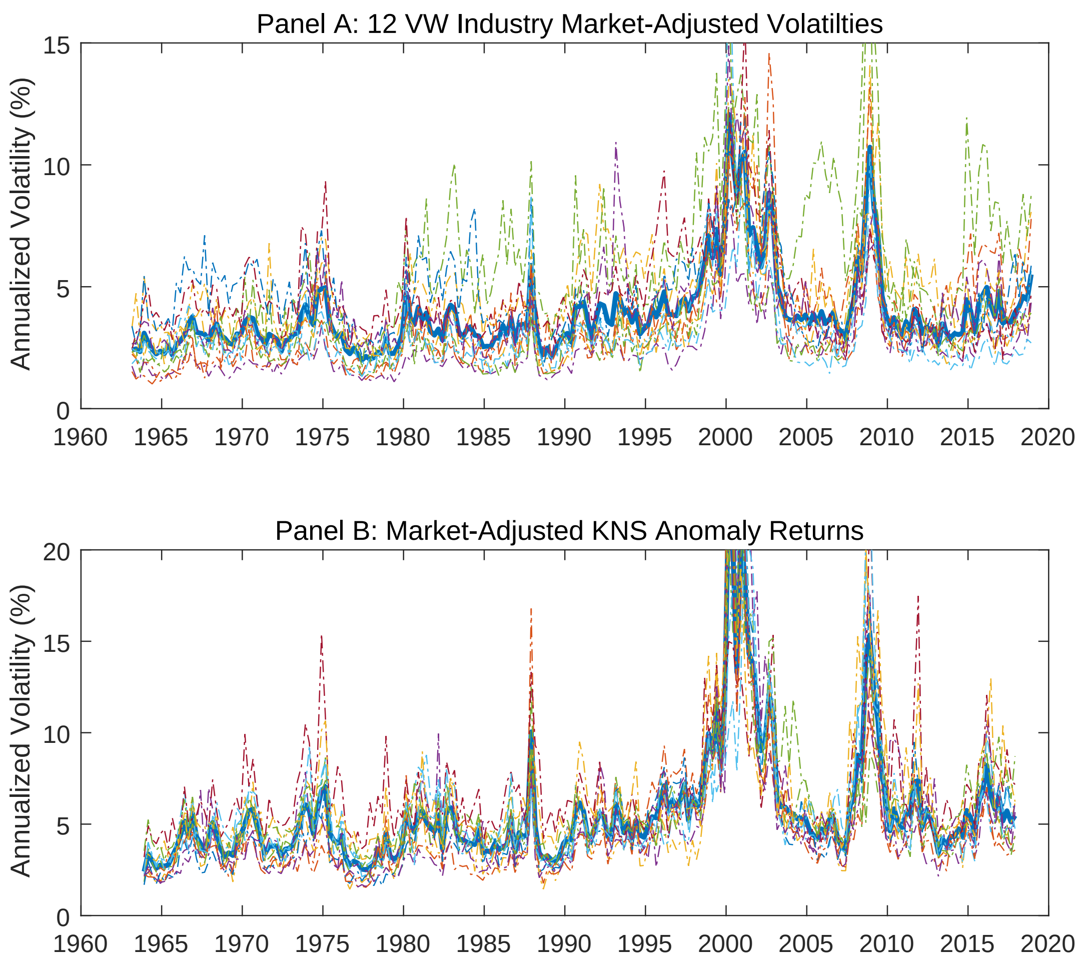


Fig. 3: A Comparison of Alternative CFV Series

This figure shows alternative versions of the common factor volatility (CFV) time series extracted from different sets of factor and anomaly portfolios. Each CFV series is constructed as the average of the residual volatility, based on a market model regression, across factor or anomaly portfolios in the corresponding set. Results are included for the full set of characeteristics-based factors, a large set of anomaly returns, 12 value-weighted industry portfolios, and a set of 10 statistical factors. The sample period is 1965–2018 quarterly.

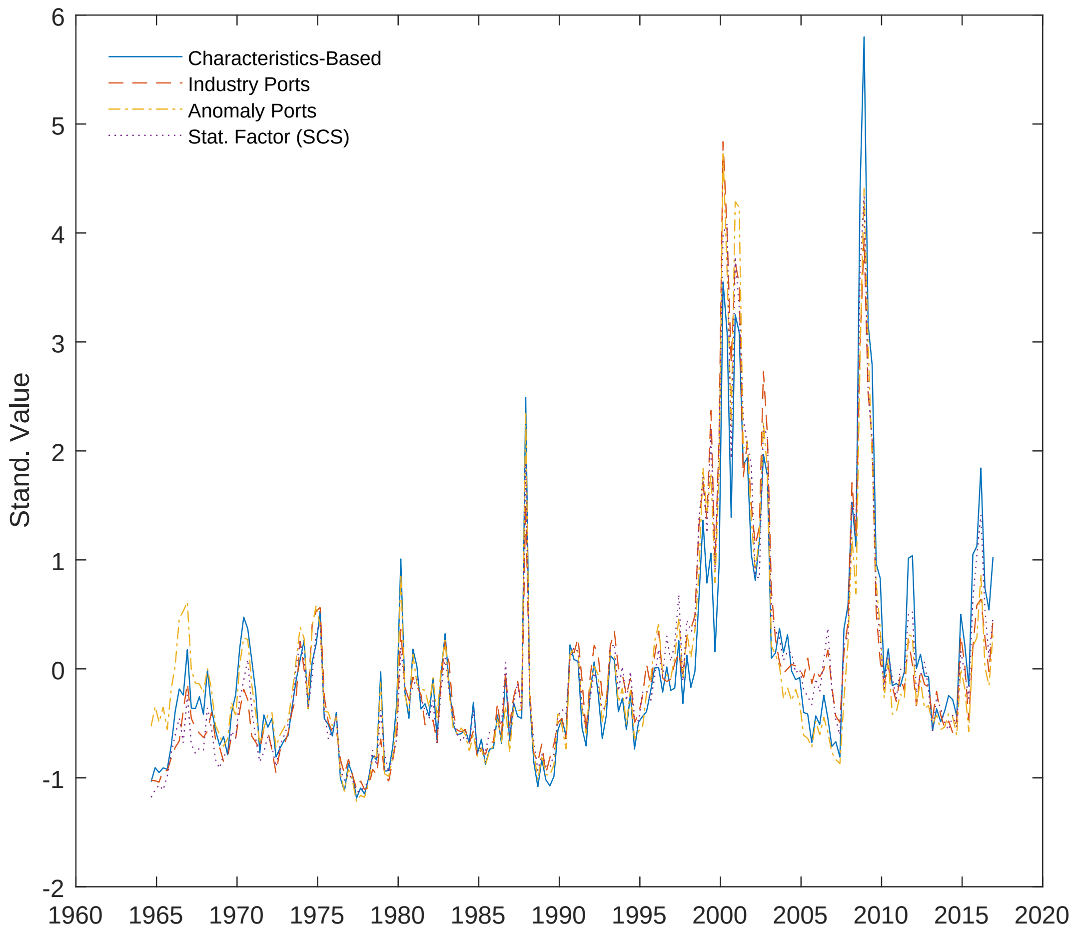


Fig. 4: Characteristics-Based Factor Correlations

This figure shows plots of raw and market-adjusted (residual) correlations for characteristics-based factors. Panel A plots quarterly time series of pairwise correlations for the following characteristics-based factors: SMB, HML, RMW, CMA, and UMD. Correlation values each quarter are computed as the sample correlation of daily factor returns within the quarter. To facilitate comparison of the dynamics of correlations across factor pairs, each pairwise correlation is shown as the de-meaned value relative to the time series average for the correlation. Panel B displays similar information, except based on the (daily) residuals of factor returns from market model regressions estimated each calendar year.

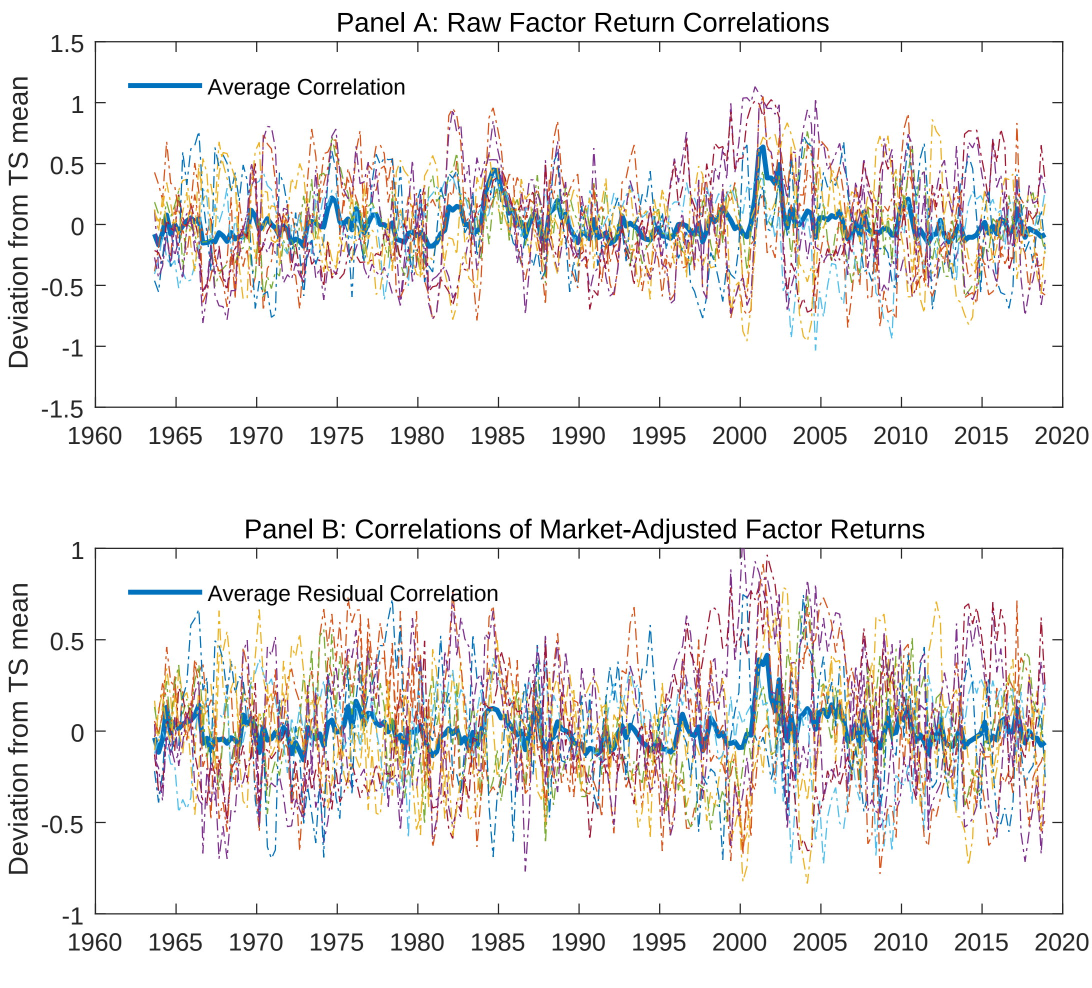


Fig. 5: Common Factor Volatility for Alternative Ranges of Statistical Factor Decomposition

This figure compares time series for common factor volatility (CFV) obtained from different sets of statistical factors. We first apply the RP-PCA approach to obtain statistical factors using an underlying set of daily portfolio returns for the GHZ anomaly portfolios and 30 industry portfolios. Statistical factor returns are normalized to average 20% annually unconditionally. The figure plots a simple measure of common volatility computed as the average of individual factor volatility series for alternative groups of 10 factors. The first group consists of the first ten factors based on the RP-PCA criterion. The next group consists of factors 10-20, the third group consists of factors 40-50 and the final group consists of factors 60-70. The figure plots the quarterly time series of common volatility for each group.

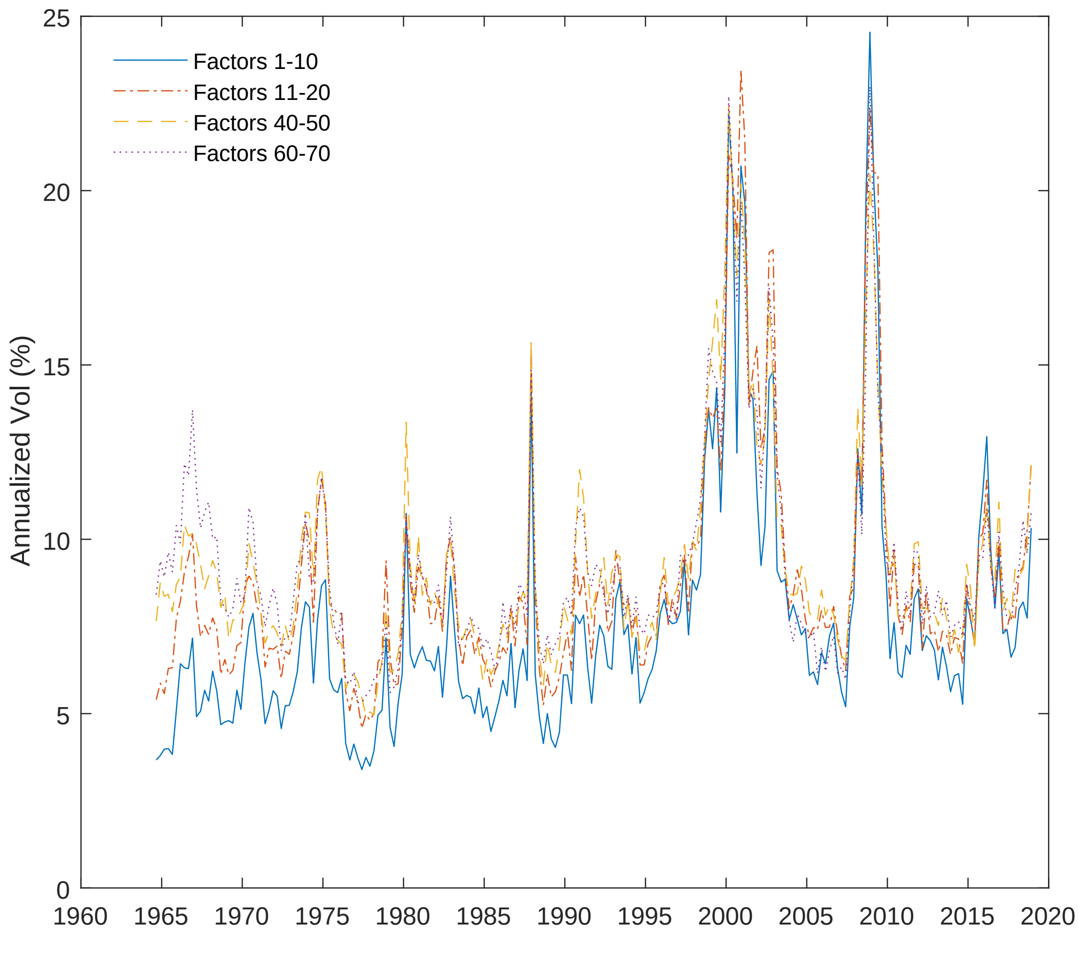


Fig. 6: Common Factor Volatility, Market Volatility, and Common Idiosyncratic Volatility

This figure compares time series for common factor volatility (CFV), market volatility (MKT VOL), and common idiosyncratic volatility (CIV). The CFV series is based on the factor set of 10 statistical factors. CIV is constructed following Herskovic et al. (2016). Market volatility is measured using the sum of squared daily returns for the Fama-French market factor. Panel A plots annual time series for each volatility measure. To facilitate comparison, Panel A plots the standardized value of the natural logarithm of each volatility measure. Panel B shows the deviations of market volatility and CIV from CFV, measured as the difference between, e.g., the standardized value of log market volatility and the standard value of the log of CFV.

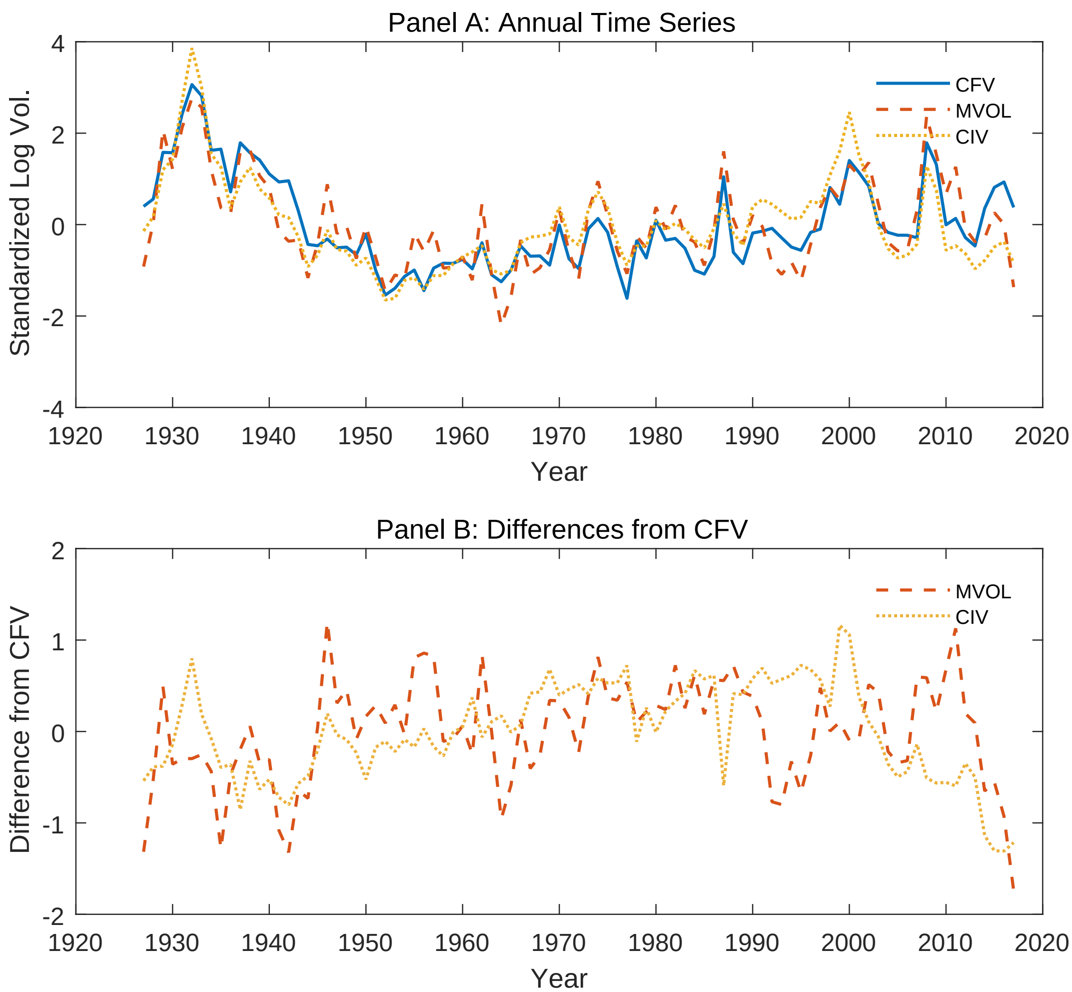


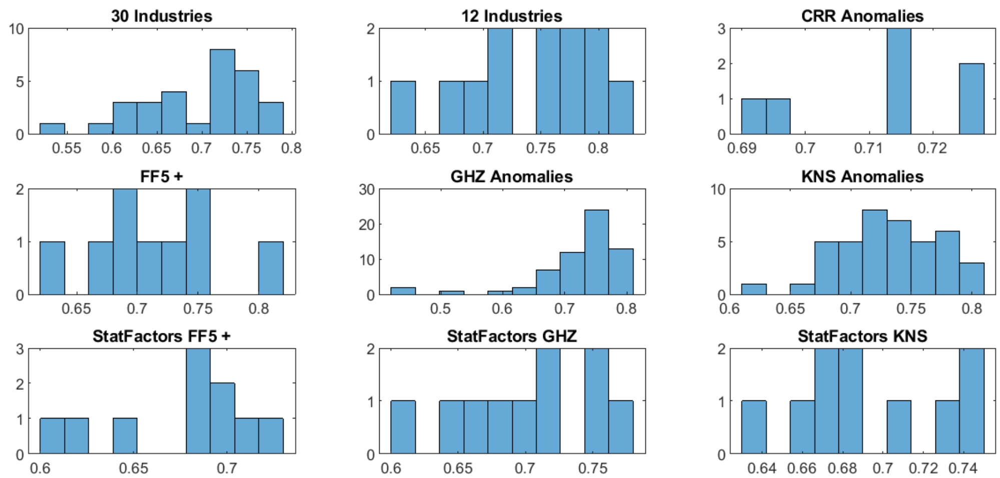
Fig. 7: CFV and Factor Vol

This figure shows histograms of betas (Panel A) and R^2 (Panel B) for sets of factors' regressions of the following form:

$$log(\sigma_{F_k,t}) = a + \beta \cdot log(CFV_t) + \epsilon_t,$$

where $\sigma_{F_k,t}$ denotes factor k's volatility in month t and CFV_t denotes the quarterly measure of Common Factor Volatility. All monthly log volatilities have been linearly detrended and normalized to have mean zero and standard deviation of 1. In all cases the sets of portfolios and dates used to generate the histogram corresponds to the portfolios described in Table 1.

Panel A: Betas



Panel B: R-Squared

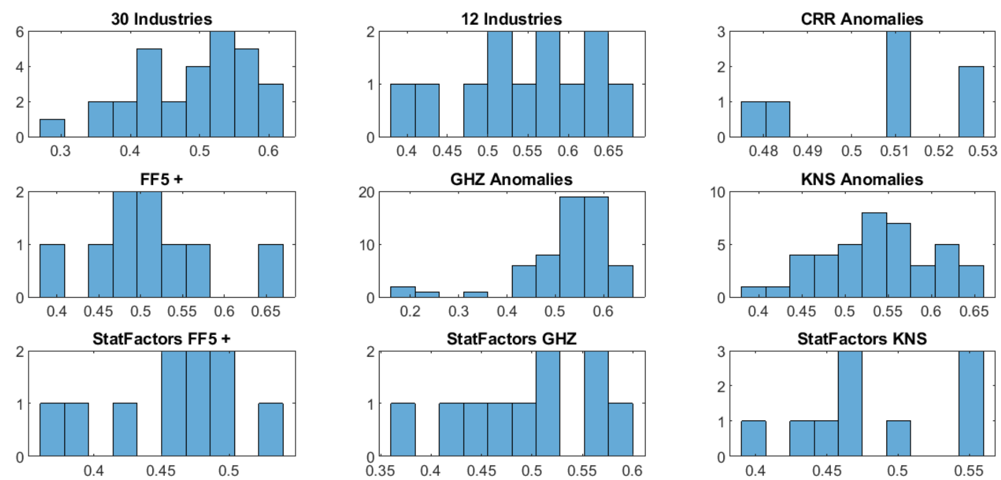


Fig. 8: Residuals Predict Factor Volatilities

This figure shows histograms of coefficient estimates and corresponding t-statistics for predictive regressions of one-month ahead volatilities using residuals and lagged volatilities of each factors' own-volatility. Residuals are taken from the full sample regression of portfolio log volatilities on log of CFV:

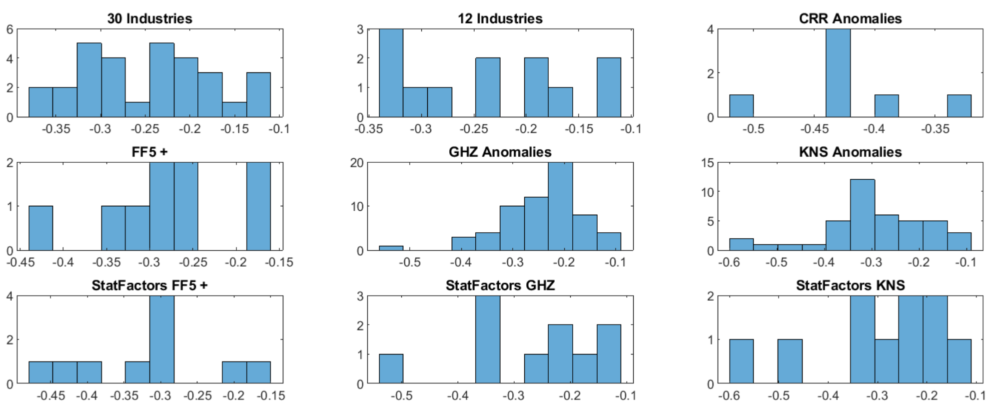
$$log(\sigma_t^k) = a + \beta \cdot log(CFV_t) + \epsilon_t.$$

We then use the residuals from the above regression to forecast future values of $log(\sigma_{t+1}^k)$ one month in the future using the following specification:

$$log(\sigma_{t+1}^k) = a + \lambda \cdot r_{k,t} + \beta \cdot \sum_{l \in H} log(\sigma_{t-l+1,t}^k) + \epsilon_t,$$

where $r_{k,t}$ denotes the residual from the regression of volatilities on CFV and $\sigma_{t-l+1,t}^k$ denotes volatility over the period $t-l+1$ through t. Using the HAR model for factor volatilities with $H = \{1, 3, 12\}$ controls for past values of each factors' past volatility. All monthly log volatilities have been linearly detrended and normalized to have mean zero and standard deviation equal to one.

Panel A: Residual Coefficients



Panel B: Residual T-Statistics

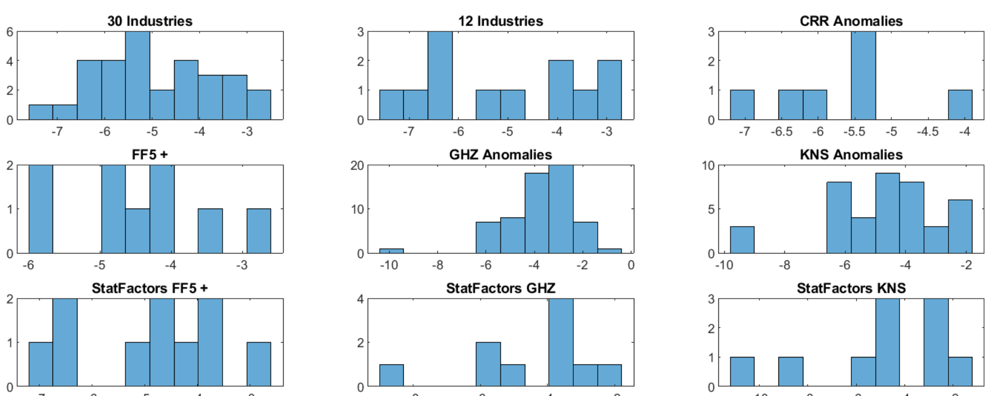


Fig. 9: The common volatility in macroeconomic factors and in their underlying series

This figure compares time series in the monthly common volatility in CRR factor mimicking portfolios ('CRR'), their underlying macroeconomic series ('CRRunder'), and 30 VW industries ('IndVW') constructed in Table 4.